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AN ESSAY

ON

MAGNETIC ATTRACTIONS,

AND

ON THE LAWS

OF

TERRESTRIAL AND ELECTRO MAGNETISM;

COMPRISING

A POPULAR COURSE OF CURIOUS AND INTERESTING EXPERIMENTS ON THE LATTER SUBJECT, AND AN EASY EXPERIMENTAL METHOD OF CORRECTING THE LOCAL ATTRACTION OF VESSELS ON THE COMPASS IN ALL PARTS OF THE WORLD.

BY PETER BARLOW,

ASSOCIATE IN THE SOCIETY OF CIVIL ENGINEERS, AND OF THE ROYAL MILITARY ACADEMY.

SECOND EDITION,

MUCH ENLARGED AND IMPROVED.

ILLUSTRATED WITH PLATES BY LOWRY.

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1823.

TO

DAVIES GILBERT, ESQ. M.P. F.R.S.

MEMBER OF THE BOARD OF LONGITUDE,

&c. &c. &c.

SIR,

THE great interest you take in all useful scientific pursuits, and particularly in those connected with Nautical Science, has made me solicitous of the honour of presenting to you this new edition of my "Essay on Magnetic Attractions." And I am willing to hope, that however defective the work may be in its execution, you will at least approve of its design, and deem it not unworthy of your acceptance.

I have the honour to be,

Sir,

With great respect and esteem,

Your very obedient

and humble Servant,

THE AUTHOR.

Royal Military Academy,
Oct. 22, 1822.

PREFACE.

The subject of the local attraction of vessels having engaged the attention of several eminent practical navigators and philosophers, I was induced in the year 1819 to undertake a course of experiments, with a view of deriving some principle of computation, or other method, for correcting this source of error, it having been generally admitted that, at this time, no rule had yet been given that could be considered applicable to the purpose in all parts of the world.

In the course of these experiments I was so fortunate as to fall upon an easy practical mode of correction, wholly independent of calculation; I also discovered certain magnetic laws which seemed to me likely to pave the way to a mathematical theory of magnetism.

These results were drawn together and published in the year 1820, under the title of "An Essay on Magnetic Attractions," and which may still be considered as the foundation of the present

work; although, having considerably extended my views, and multiplied my experiments and investigations, I have found it necessary to change, in a slight degree, the former title, whereby to comprehend the science of Electro Magnetism, which forms one of the divisions of the present volume.

The leading object of this edition, however, as of the former, is the developement of the mathematical principles of magnetism, and their application to the correction of the local attraction of vessels, which is of more and more importance, as every year is leading to some new application of iron in the construction and equipment of ships of war, and which, if persevered in without some mode of correction, would soon render the compass worse than useless as a nautical instrument.

It may be observed, for example, that besides there being at present considerable more iron ballast than formerly, the water casks are now replaced by iron tanks presenting an immense attracting surface; iron knees, sleepers, plates, and in some cases the riders, have been introduced in lieu of those of timber; even the hempen cables have been put *hors de service* by the patent

cables of iron,—gun-carriages of this metal are at this moment supplanting those of the usual material: the ingenious patent capstan of Captain Phillips, which will doubtless soon become generally applied, is principally of iron, and although of no considerable mass, is so situated as to affect the compass very sensibly; and lastly, it seems probable that even the masts* are to be attempted in this material.

The work is now divided into three parts; the first containing the greater portion of the matter delivered in the former edition, with some additional experiments, which by the favour of my Lords Commissioners of the Admiralty, I have been enabled to make on board several of His Majesty's vessels, and the results obtained by a series of observations in H. M. S. LEVEN during a voyage of sixteen months. I have also added to this part the results of a series of experiments made with a view of ascertaining the effect which the iron of a vessel has upon the rates of chronometers; and another series on the comparative magnetic power of different kinds of iron and steel; and on the

* See Addenda.

effects of heat in changing the magnetic power of iron bodies.

The second part is theoretical, in which I have endeavoured to show that all the laws deduced in the first part from experiment only, are the necessary consequence of a certain hypothesis, exceedingly simple in its principles, and general in its application. The formulæ deduced in this part are shown to be easily convertible into others which embrace all the known laws of terrestrial magnetism; and an attempt is made to compute the annual change in the inclination and declination of the needle.

The third part is appropriated to the science of Electro Magnetism, which has had its birth since the former edition of this work was published. In the first section is given an historical sketch of the present state of this new science; in the second are described the experiments I have been led to make with a view of reducing its laws to mathematical principles; and in the third is given a course of interesting experiments, due to the several ingenious philosophers who have interested themselves in this pursuit; and in which I have

endeavoured to show their mutual dependence on each other, and their general agreement and particular connection with the mathematical theory advanced in the second section.

I am not aware that it is necessary to enter in this place into a more minute analysis of the contents of the present volume; but I cannot conclude this Preface without expressing my grateful acknowledgment for the generous and flattering manner in which the former edition of this work was received, both by the nautical profession and by the most distinguished philosophers and mathematicians. I am also highly indebted to my Lords Commissioners of the Admiralty, and to the Honourable the Principal Officers and Commissioners of His Majesty's Navy, for the various facilities I have experienced in carrying my experiments into execution; and I am willing to flatter myself that these public Boards will feel satisfied that I have made the best use in my power of the opportunities I have thus enjoyed.

Royal Military Academy,
Oct. 22, 1822.

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ERRATA.

Page 12 line 19, *for* Lecourt *read* Lecount
19 last line of the note, *for* the inclination *read* tan. of the inclination
22 line 4 from bottom, *for* distance *read* distances
24 line 14, *for* there *read* therefore
84 line 5 from bottom, *for* c *read* C
171 line 15, *for* number of vibrations *read* times of vibration
172 line 2 from bottom, *for* cos *read* cos *l*
273 line 16, *for* plate 5 *read* plate 4

AN

ESSAY

ON

MAGNETIC ATTRACTIONS, &c.

PART I.

Containing a Detail of Magnetical Experiments.

SECTION I.

PRELIMINARY OBSERVATIONS AND EXPERIMENTS.

1. I FIRST undertook the experiments, which form the basis of the following Essay, with a view of finding some method of correcting the local attraction of a ship's guns, and other iron on the compass, and it accordingly formed the prominent object in the former edition of this work. In the present instance, I have attempted something more, but I have still preserved the original form and arrangement, as far as the matter has been retained, and have only added such chapters, and made such alterations as have been rendered necessary, from subsequent experiments and investigations.

When I begun my experiments, the laws of magnetic attractions were unknown; the doctrine of magnetism consisted merely of a mass of detached experimental facts, which were, indeed, in some instances connected with each other by means of certain hypotheses, as we still endeavour to generalize the phenomena exhibited by an electrical machine; but in the former case as in the latter, no successful attempt had yet been made to reduce the results to the dominion of analysis.

This then was the task which I proposed to myself, when I undertook the following experiments, and which my situation, at Woolwich, enabled me to pursue upon a very extended scale, as I could procure with facility, not only very considerable masses of iron, but those of the most uniform shape; viz. balls and shells of every denomination, from those of a pound weight to others of half a ton, and from whose regularity of figure I had every reason to expect such a series of results as would, in all probability, enable me to detect the general principle of action.

Having, however, no similar operations to refer to, my first steps were necessarily slow, and in some instances useless, many of which I shall therefore omit the recital of in this section. At the same time, in order that the reader may see the views by which I was in a great measure guided, I propose to take a glance of the whole course, from my first

attempt on balls of a few pounds weight, to the gun of 58 cwt., and my deduction with reference to the most irregular masses, and to ships of every denomination.

2. *First course of Experiments.*—In these I began by describing on a level platform several concentric circles, from 8 inches to 16 inches radius, drawing through the centre a line, in the direction of the magnetic meridian; I then set off my east and west points; and, lastly, divided the whole circle into equal parts of 10 degrees each.

Having now adjusted my compass over the centre of the circle, I applied successively at every point of division, and at different distances from the centre, the shells belonging to the 5½ inch, 8 inch, and 10 inch howitzers, weighing respectively 14lbs., 48lbs., and 96lbs., and noted the deviation caused by each in every position in which it was placed; expecting thus to be able to draw some law, or to establish some relation between the masses, distances, and positions of the shells, and the deviation of the needle. In this, however, I was disappointed, and it would therefore be useless to give these results at length. It will be sufficient to observe, that the general effect was to produce a deviation of the north end of the needle towards the west, while the shell was passed from the north through the east to the south; and then an opposite deviation, while it was carried

through the other semicircle; the greatest deviation being in all cases when the shells were situated between the south and east, or the south and west points of the circle.

3. Although the deviation, being all in one direction during the motion of the shell in each semicircle, was contrary to what I had anticipated; yet I did not consider the circumstance as very extraordinary, till I applied a small solid ball of two inches diameter. I was then much surprised to find that this caused a directly contrary effect; that is to say, it produced first an eastern deviation, which arrived at its maximum between the north and the east; it then decreased to zero, between the east and south: after which, the deviation became west, attained its maximum, and again vanished when the ball was due south.

The same happened again in the contrary order, while the ball was passing from the south through the west to the north. I could perceive no cause for this apparent anomaly, and was, at length, obliged to leave it as an experimental fact, at that time wholly inexplicable.

4. I now proceeded with my experiments on the shells, exactly in the same manner as above stated; except that I raised the compass in the centre to the level of the equator of each ball respectively, when I again observed a similar result to that above mentioned; that is, in this instance I found

first an eastern deviation, which vanished when the ball was due east; then a western deviation, while the ball was passed from east to south; and the same in the contrary order while passing from south to west, and from west to north. After this, raising the compass ten inches from the platform, I found, instead of the deviation being first western, as in my leading experiments, or first eastern and then zero and western, as in the last, that it was wholly eastern in the first semicircle, and western in the second: the results in this case were therefore exactly the reverse of those in the first instance.

5. It was thus rendered obvious, that the deviation, both as regards its quantity and direction, depended upon the position of the centre of the ball with respect to the compass. In fact, I found hat at every point, except the north and south, if I moved the ball from above downwards in the same vertical, I produced first an eastern and then a western, or first a western and then an eastern deviation: consequently, in each of those verticals there must be some one point in which the deviation was zero; that is, in which the matter of the ball had no effect. These points of no action, I conceived, would in all probability be found posited in one plane, and my next object was therefore to ascertain whether this was the case; and if so, to determine its inclination to the horizon, for it was obvious that it was not parallel to it.

6. With this view, I had a strong table made with copper fastenings, fixed with its feet in the ground, and rendered tolerably secure, and in its centre a circular hole, a little more than ten inches in diameter, was formed, through which, by means of a block and pulley, the ten inch shell could be raised and lowered at pleasure. The upper surface of the table, which was covered with paper, was, as in the former instance, divided according to the points of the compass; the magnetic meridian being first accurately determined. The compass being now carried round the ball, instead of the latter being passed round the former, as in the first experiments.

This being prepared, I elevated the ball till its action was imperceptible; and then gradually lowering it, I noted the deviation at various altitudes of the ball, with the compass at each point of division of the circles, but more particularly in that of eight inches; observing also very accurately the height or depth of the centre of the shell above or below the pivot of the needle, when the deviation was zero. These last results, indeed were the only ones applicable to my present inquiry; and from these I ascertained that the several points of no action were (as I had suspected) all situated in one plane; the inclination of the plane itself to the horizon being found nearly equal to 20 degrees, declining directly from the magnetic north point to the south.

This plane is therefore either exactly, or very nearly perpendicular to the direction of the dipping needle.

The cause of the apparent anomaly which I had observed, relative to my small solid ball, now became obvious; for the centre of this ball being but a little above the pivot of the needle, the plane of no attraction passed above or below that pivot, according to the position of the former; whereas in the larger shells it passed above it in all their positions, except when the compass was elevated, and then the same happened in these cases as in that. (See Art. 3, &c.)

I omit the detail of the experiments and computations here referred to, as they will be found repeated in a subsequent section, with a larger ball and with a more perfect apparatus.

7. Having established this point, I could not but consider it as an important step towards attaining the object I had in view; and I wished therefore to improve the construction of my apparatus, and to employ a ball or shell of larger dimensions. I accordingly procured from the Royal Arsenal a solid thirteen inch ball, such as is employed in proving the mortars of that dimension, weighing 288lbs.; and having ordered a new table, and procured a better system of pulleys, I repeated my former experiments, and confirmed the results I

had drawn from them, as far as related to the inclination and direction of the plane of no attraction.

Being thus assured that there are in every ball of iron two planes, in which the compass may be any where posited, without being influenced in its direction; the one that of no attraction, as stated above, and the other the vertical plane, corresponding to the magnetic meridian; my next object was to ascertain how far the angle of deviation of the needle was influenced, and what law that deviation observed when the compass was removed out of those planes. But before I proceed to describe the experiments performed with a view to this determination, it may not be amiss to examine the deductions already made, which may be stated as follows:—

8. In the first place, it has been shown that every iron ball has what (from analogy to the case of terrestrial magnetism) may for the present be denominated a *magnetic equator*, lying in the plane of no attraction, above mentioned.

9. That like the earth, also, it may be supposed to have two magnetic poles; the one directed towards the north, and the other towards the south; the line joining those poles being parallel to the natural magnetic direction of the dipping needle.

10. These experiments likewise seem to indicate that the effect produced upon the needle by the iron, (the distance being the same,) depends entirely

upon the position of the centre of the ball, with reference to the pivot of the needle, and not to its position with regard to either extremity.

11. Having made these deductions, I conceived an ideal sphere to be circumscribed about the ball of iron; and assuming the circle of no attraction as an equator, and the poles of that circle as the poles of the sphere, I imagined circles of latitude and longitude to be described upon it, and wished, if possible, to pass the compass round the ball in these several circles, keeping it always at the *same distance* from the centre; so that, in taking the deviations, I might separate the effect due to position, from that which might otherwise have arisen from a change in the distance. I determined also, in order to disengage the effect due to the longitude from that which had a reference to the latitude, to pass the compass in the first place over circles of latitude only, viz. in circles perpendicular to the magnetic equator: and finding after a few trials what I had indeed anticipated, that the deviations were the greatest in that circle which passed from the poles through the east and west points of the equator, I made this my first or principal meridian, and considered its longitude as zero.

12. My plan being thus laid, and my table now divided in equal parts of five degrees, and these again in certain places subdivided into less portions, I began by computing how much the centre of the

ball ought to be raised or depressed when my compass stood on any given division, and how much the compass ought to be approximated towards the centre of the circle, in order that the former might fall upon the surface of the imaginary sphere; and then again what was the latitude of its position.

These, as is obvious, required the solution of so many spherical problems, in which there were given the angle on the plane of the table from the magnetic east or west points of the horizon, which may be considered as the base of a spherical right angled triangle, and the angle at the base, equal to the natural dip of the needle 70° 30′, to find the hypothenuse and perpendicular. The former gave me the point in the circle in which the compass would be placed, or its latitude; the sine of the latter to any given radius, equal to the proposed distance, the height or depth of the centre of the ball above or below the plane, passing parallel to the horizon through the pivot of the needle; and the cosine of the latter to the same radius, the distance of the compass from the centre of the table.

13. Having prepared myself with these numbers, I began my series of observations with three different compasses, noting very carefully the deviation due to each position, limiting myself principally to the circle of twelve inches radius: and having thus obtained a very extensive series of results, I made

various comparisons between the trigonometrical lines of the angles of deviation, and those of latitude; and after a few fruitless attempts I succeeded in deducing the following law; namely, *that the tangents of the deviations are proportional to the rectangle of the sine and cosine of the latitude, or to the sine of the double latitude.*

I omit giving the particular results of these experiments, for the reasons already assigned; namely, that they were afterwards repeated in a more accurate manner, the detail of which will be found in a subsequent article.

Having thus established the law of deviation as it depended on the latitude, I made a very few experiments with a view to a similar determination in reference to longitude; from which I concluded (though not without some hesitation), that, all other things being the same, the deviation was proportional to the cosine of the longitude; and with this deduction I concluded my first series of experiments, finding, that notwithstanding the improvement I had made in my apparatus, it was still not sufficiently accurate for pursuing my inquiries according to the more extended view I had now formed on the subject.

14. I had arrived at the above conclusions about the beginning of May, 1819, while the vessels designed to explore a north-west passage were still in the river; and knowing that the attention of

Captains Ross and Sabine had been in their late voyage a great deal engaged on the subject of local attraction, and considering these deductions as bearing strongly upon that inquiry, I wrote to Captain Sabine, stating the result of my investigations, and expressing my wish that he would endeavour to ascertain whether the plane of no attraction (which doubtless exists in all latitudes) was every where inclined to the horizon at an angle equal to the complement of the dip, or at least whether it was so in those parts which he might visit; to this letter I received an answer, which led me to hope that he would have been able to decide the question: but on the return of the discovery ships here alluded to, I found that Captain Sabine had not had an opportunity of trying these experiments; my deduction, however, has been very satisfactorily confirmed by Mr. Lecourt, by a long series of experiments from the Cape of Good Hope to England, of which a more particular notice will be found in a subsequent page of this Essay.

15. Soon after the above communication to Captain Sabine, having drawn up a statement of my experiments and deductions, I transmitted it, through the favour of General Mudge, to the Royal Society; and it was accordingly read at the meeting of the 20th of May, 1819, and I was afterwards informed that it had been deposited in

the archives of the society, and that I could not have it returned, nor a copy of it, without paying for its being transcribed. Not being desirous of complying with the latter condition, particularly as it might occasion some delay, I drew up the preceding statement, principally from memory and the few rough notes I had by me: I had, indeed, the detail of my experiments; but, for the reason before assigned, I have purposely refrained from giving them.

My principal reason for stating the above particulars, is to make the circumstance an apology for any want of order and arrangement that may be apparent in the preceding and following sections; for the latter having been all written upon a supposition that it would appear with the former, I had not been so explicit in some points, as became necessary after I found it was to be published without its precursor; and I have therefore been obliged to introduce explanations in certain parts that were before mere matters of reference: this may occasion some want of order and perspicuity; but these cases, I hope, will be but few.

SECTION II.

SECOND COURSE OF EXPERIMENTS; DESCRIPTION OF APPARATUS, INSTRUMENTS, &c.

16. Having explained, in the preceding articles, the general purpose of my experiments, and the nature of the deductions I had already drawn from them, I shall without farther introduction proceed to the description of the present series, which are nearly of the same kind as the foregoing: in fact, the only difference is, that my apparatus are here of a more accurate construction, the instruments more perfect, and the experiments performed with them, more varied and extensive.

Description of the Apparatus.—The table described in my former paper being found too small for carrying on my operations to the extent which seemed desirable, and as the place in which it was erected on my own premises, was not so convenient as I could have desired, General Mudge, who had taken considerable interest in the subject, very readily complied with my request, to be allowed the model room belonging to the Royal Military Academy, for the further pursuit of my inquiries; at the same time giving me permission to have such an apparatus constructed as seemed best to answer my intended purpose. I accordingly ordered a very

firm and solid round table to be made, 4 feet 8 inches in diameter, and 3 feet 2 inches high. In the centre of this table, a hole 13¾ inches in diameter, was cut, for the purpose of allowing the ball to pass freely through it. A piece of wood of the same thickness as the top of the table was made to fit this hole very accurately, and which might therefore be removed, or employed, as occasion required, three buttons being placed under the table near the edge of the hole for the purpose of resting it upon when employed, and which were turned back when it became necessary to pass the ball through.

In order now to prevent any shaking or trembling in the needle, four holes were cut in the floor, and piles driven into the earth below, to set the feet of the table upon, which effectually prevented any inconvenience of that kind, and the whole was rendered perfectly steady and horizontal. The centre piece being now fixed in its place, and a small brass centre sunk in it, to prevent the galling of the hole; the plane of the table was very accurately divided into equal parts of $2^{\circ}\frac{1}{2}$ each, by lines passing from the centre to the circumference, and all parting from one principal diameter, drawn in the direction of the magnetic meridian.

The table being thus prepared, a *gin* consisting of three poles twelve feet high, copper mounted

and tied, was erected over the centre of the table; and from the vertex of the former the ball was suspended by a system of pullies on Smeaton's principle, the power of which was as 12 to 1, whereby the purchase was reduced from 288lbs., (the weight of the ball) to $\frac{288}{12}$=24lbs.; consequently the raising or depressing of the latter became easily manageable without a second person to assist.

17. *Description of the Instruments.*—The compasses which I had employed in my former experiments could only be read off to quarters of degrees, which therefore, although very perfect in their construction, I wished to replace by some one which might be read to minutes. My friend, Francis Baily, Esq., knowing that Mr. Arrowsmith, the celebrated geographer, had an excellent instrument of this kind, which had been made under the directions of the late Dr. Lorimer, he introduced me to the former gentleman, who very politely entrusted his instrument to my charge. The needle of this compass was six inches in length, of the bar form, and very powerful; it was suspended in a brass box seven inches in diameter, the circle graduated to half degrees, which together with the vernier carried on the north end of the needle, enabled us to read off very accurately to minutes.

18. I have been particular in describing this

instrument, not because I made much use of it in my experiments, but because I found a defect in it, which may probably more commonly appertain to compasses of this description than is usually imagined, and which I conceive is important to mention.

Having, immediately after my apparatus was erected, repeated, with the above instrument, a few of my former experiments, I found myself considerably perplexed with certain anomalies and irregularities, which I could not account for on any principle; till at length it occurred to me, that they were precisely what would take place, if any part of the brass box itself had become magnetic; and on trial I found this actually to be the case; for, on removing one of the pieces of brass attached to the box for the purpose of setting the instrument and fixing the sights, I found it to be strongly magnetic, sufficiently so to produce a vibration of the needle (when applied outside the glass) of 14 or 15 degrees, and to retain the same 1½ degree out of its natural direction, and the lighter needles belonging to my other compasses were drawn and retained by the same piece of brass 4, 5, or 6 degrees from their true magnetic bearing, although applied outside the glass, and therefore at nearly a quarter of an inch from the extremity of the needle. This piece of brass was by far the most powerful in

Synonymous

its effects; but still every screw and attached piece in the instruments had acquired the same quality to a certain degree, so that no dependence could be placed upon the needle except when these were all removed, which rendered its application inconvenient; I was therefore reluctantly obliged to discontinue the use of this fine instrument, and to have recourse to those employed in my former experiments.

Besides the above instrument of Mr. Arrowsmith, the late Mr. Berge, very obligingly, on the application of Major Colby, favoured me with the loan of an excellent *dipping-needle* constructed by Nairne, exactly corresponding with the description of that made by the same artist for the Hon. H. Cavendish, as given by the latter gentleman in the Philosophical Transactions for 1776, to which I beg therefore to refer. It was on this instrument I made the experiments on the dipping-needle described in the following section.

SECTION III.

EXPERIMENTS MADE WITH A VIEW OF DETERMINING THE INCLINATION OF THE PLANE OF NO ATTRACTION.

19. It being rendered obvious by my former experiments, that the laws of attraction depended

principally upon the position of the iron or compass, with reference to the circle denominated, in Section I. the *magnetic equator*: my first object was with the new apparatus, to repeat my former experiments, employing now a radius, or distance, of 20 inches, in lieu of that of 12 inches, which I had adopted in my former experiments.

The compass, with this view, was placed successively on every second division of the table; viz. at every 5° in the entire circumference, keeping its centre in every case, exactly upon the circumference of the circle of 20 inches radius. The ball was then gently raised or depressed till the needle had attained exactly its proper magnetic bearing, when the height of the centre above, or its depth below the pivot of the needle, was accurately measured, and the inclination of the plane corresponding to each position of the compass computed, by means of the formula

$$\tan I = \frac{h}{r \cos a} = \frac{h}{r} \sec. a^*$$

where I denotes the inclination, r the radius of the circle, which in the present instance was 20

* Or by the proportion,
As the radius of the circle (20 inches)
To the height or depth of the centre.
So is the secant of the angle from the east or west points,
To the inclination of the plane.

inches, h the observed height or depth of the centre of the ball, and a the angle from the north or south point of the circle. To facilitate the computation, I have, in the following table, taken the mean of the four heights or depths due to the corresponding similar positions, from the east or west points.

Experiments.

Position of Compass or value of a.	Observed Heights and Depths.				Mean value of h.	Inclination computed by the formula. $\tan I = \frac{h}{r} \sec. a$	
	From North towards East. *Depth.*	From North towards West. *Depth.*	From South towards East. *Height.*	From South towards West. *Height.*			
5°	7·10	7·00	7·05	7·05	7·05	19°	29′
10	7·05	7·00	6·95	7·00	7·00	19	34
15	6·80	6·80	6·80	6·80	6·80	19	23
20	6·65	6·70	6·55	6·50	6·60	19	21
25	6·40	6·40	6·40	6·40	6·40	19	27
30	6·05	6·10	6·10	6·15	6·10	19	24
35	5·80	5·80	5·80	5·80	5·80	19	30
40	5·50	5·40	5·40	5·50	5·45	19	11
45	5·00	5·00	5·00	5·00	5·00	19	28
50	4·55	4·50	4·50	4·45	4·50	19	18
55	4·07	4·05	4·06	4·06	4·06	19	27
60	3·57	3·52	3·52	3·55	3·54	19	26
65	3·00	3·00	3·00	3·00	3·00	19	35
70	2·35	2·37	2·37	2·35	2·36	19	01
75	1·85	1·80	1·85	1·82	1·83	20	26
80	1·25	1·25	1·22	1·20	1·23	19	30
85							
				Mean Inclination.		19	24

20. We have thus 19° 24′ for the inclination of

the circle of no attraction, which approaches very nearly to the complement of the dip of the needle, as determined by Captains Kater and Sabine in the Regent's Park, who found it to be 70° 34′, April 13, 1818. Having, however, the opportunity of ascertaining the dip correctly for Woolwich, on the needle so obligingly entrusted to my care by Mr. Berge, I made the following experiments, with a view to this determination.

		Mean time of performing 100 vibrations.*
Mean result of ten trials, face to the East	70° 35′·1	8·25 min.
Mean result of ten trials, face to the West	70° 25′·9	8·25 min.

Poles Inverted.

Mean result of ten trials, face to the East	70° 28′·2	8·19 min.
Mean result of ten trials, face to the West	70° 32′·6	8·21 min.

Mean of the above results 70° 30′·45 for the dip at Woolwich, July 13, 1819.

The extremely near approximation of these several results towards the complement of the angle obtained for the inclination of the circle of no attraction, shows that an obvious relation subsists between the two angles, viz. that the one is the complement of the other, and at the same time

* First arc of vibration 70°.

renders it highly probable that this law obtains in every part of the globe.*

SECTION IV.

DETERMINATION OF THE LAW OF ATTRACTION IN REFERENCE TO THE LATITUDE.

21. Having thus determined, by a new series of experiments, the position of the circle of no attraction, which I shall henceforward, as in the preceding series, call the *magnetic equator;* my next object was to determine the law of attraction out of this circle, and first in the circle S E N, drawn perpendicular to the equator QQ′, and passing through the east and west points of the horizontal circle H R. I have already explained

* These results have been since verified by Mr. Christie, on the same apparatus which I employed. He proceeded as follows: Having assumed the inclination of the plane of no attraction to the horizon, at 19° 30′, he computed the heights at which the ball ought to be found, when the disturbing force on the needle should cease, and having then actually observed the same, he found the computed and the experimental situation to agree very nearly with each other; not differing more than $\frac{1}{20}$th of an inch for distance, which were not less than 14 inches. See Mr. Christie's Memoir on this subject, in Part I. of the Transactions of the Cambridge Philosophical Society.

my motive for selecting this circle, the computations I employed, and the method I pursued in order to carry the compass round the ball in it: but it may not be amiss to be a little more explicit, and to illustrate what I have before stated, by means of a diagram.

22. Let H Z R fig. 1. represent a sphere concentric with the iron ball C; N,S, the north and south poles, with reference to Q Q′ the equator, or circle of no attraction; Z the zenith of the sphere, H R a circle parallel to the horizon, and S E another circle passing through the magnetic east and west points of the horizon, where it also meets QQ′: imagine also the quadrant Z L V to be drawn to any point V in the horizon, cutting S E in some point L; from L let fall the perpendicular L M, which will meet the plane of H R in the line joining V with the centre C.

Now, the arc E V being supposed given, we may readily compute the point L, where the arc S E is intersected by the quadrant Z V, and then the arc L E will be the latitude of that point, with reference to Q Q′ as an equator; and the line L M and C M, (the sine and cosine of the arc V L, to any assigned radius,) will show how much the compass ought to be elevated above the centre of the ball, and at what distance it ought to be placed from the centre of the table, to correspond with that point L.

The formulæ for these computations are as follow; viz. In the right angled spherical triangle V E L, we have given, the right angle at V, the angle L E V, and the arc E V; to find the hypothenuse LE, or the latitude, and the side or perpendicular L V.

For the former we have

$$\tan LE = \tan VE \cdot \text{see } VES \text{ *}$$

and for the latter

$$\tan VL = \sin VE \cdot \tan VES$$

from the sine and cosine of which latter arc, the values of L M and C M are readily determined for any proposed radius.

We have there only to assume for V E, the several arcs or divisions of the table, viz. $2\frac{1}{2}°$, 5°, $7\frac{1}{2}°$, 10°, &c.; and for the angle V E S the dip of the needle (which for simplicity I have taken 70° 30′) and all the several particulars stated above are already found, as in the following table, except the numbers in the last column, which are deduced

* Or which is the same,
As radius
Is to tangent of arc V E
So is, secant of angle V E S
To the tangent of arc L E,
and As radius
Is to the sine of arc V E
So is tangent of V E S
To tangent of arc V L.

from the empyrical law, derived from the first series of experiments; namely, that the tangent of the deviation of the needle is proportional to the sine of the double latitude; or, which is the same, the sine of the double latitude divided by the tangent of deviation is a constant quantity, the longitude being zero.

EXPERIMENTS.

23. *In the Circle S E, of which the Longitude is* 0°. *Ball* 288*lbs.*

RADIUS OF CIRCLE 12 INCHES.

Position of Compass.	Latitude.	Longitude.	Height of centre.	Distance from the centre of the table.	Deviation of compass East.	Deviation of compass West.	Mean deviation.	Ratio of sin. 2λ* / tan. Δ
			inches.	inches.				
2° 30′ E. or W.	7° 27′	0° 0′	1·465	11·91	10° 0′	10° 15′	10° 7½′	1·439
5 0	14 41	ditto	2·87	11·65	19 30	19 45	19 37½	1·375
7 30	21 31	ditto	4·15	11·26	26 30	26 30	26 30	1·369
10 0	27 51	ditto	5·28	10·77	32 0	31 30	31 45	1·335
12 30	33 35	ditto	6·26	10·28	34 15	34 0	34 7½	1·365
15 0	38 45	ditto	7·08	9·68	35 45	35 30	35 37½	1·363
17 30	43 22	ditto	7·76	9·15	36 15	36 15	36 15	1·362
20 0	47 28	ditto	8·33	8·63	35 30	36 0	36 15	1·328
25 0	54 24	ditto	9·20	7·76	34 0	35 0	34 30	1·378
30 0	59 58	ditto	9·79	6·93	32 15	32 30	32 22½	1·367
35 0	64 30	ditto	10·21	6·30	30 0	29 45	29 52½	1·353
40 0	68 18	ditto	10·51	5·79	26 45	26 45	26 45	1·363
50 0	74 21	ditto	10·88	5·03	21 0	21 0	21 0	1·354
60 0	79 6	ditto	11·10	4·54	15 15	15 15	15 15	1·362
70 0	83 4	ditto	11·22	4·23	10 0	10 0	10 0	1·359
80 0	86 38	ditto	11·30	4·05	5 0	4 45	4 52½	1·375
							Mean ..	1·365

* λ here denotes the latitude, and Δ the angle of deviation from the magnetic north.

24. *In the Circle S E, of which the Longitude is* 0°. *Ball* 288*lbs.*

RADIUS OF CIRCLE 15 INCHES.

Position of Compass.	Latitude.	Longitude.	Height of centre.	Distance from the centre of the table.	Deviation of compass East.	Deviation of compass West.	Mean deviation.	Ratio of $\frac{\sin. 2\lambda}{\tan. \Delta}$
			inches.	inches.				
2° 30′ E. or W.	7° 27′	0° 0′	1·83	14·88	5° 15′	5° 0′	5° 7½′	2·867
5 0	14 41	ditto	3·58	14·56	10 15	10 0	10 7½	2·746
7 30	21 31	ditto	5·19	14·07	14 0	14 15	14 7½	2·712
10 0	27 51	ditto	6·60	13·47	16 45	17 0	16 52½	2·724
12 30	33 35	ditto	7·82	12·79	18 30	18 45	18 37½	2·736
15 0	38 45	ditto	8·85	12·11	19 30	20 0	19 45	2·719
17 30	43 22	ditto	9·71	11·43	20 0	20 30	20 15	2·706
20 0	47 28	ditto	10·42	10·79	20 0	20 15	20 7½	2·719
25 0	54 24	ditto	11·50	9·65	19 0	19 30	19 15	2·711
30 0	59 58	ditto	12·24	8·67	17 30	17 45	17 37½	2·728
35 0	64 30	ditto	12·76	7·93	16 0	16 0	16 0	2·710
40 0	68 18	ditto	13·14	7·24	14 0	14 10	14 5	2·732
50 0	74 21	ditto	13·60	6·30	10 30	11 0	10 45	2·736
60 0	79 6	ditto	13·88	5·67	7 30	7 30	7 30	2·821
70 0	83 4	ditto	14·03	5·29	5 0	5 0	5 0	2·739
80 0	86 38	ditto	14·12	5·07	2 30	2 30	2 30	2·685
							Mean ..	2·737

25. *In the Circle S E, of which the Longitude is* 0°. *Ball* 288*lbs.*

RADIUS OF CIRCLE 18 INCHES.

Position of Compass.	Latitude.	Longitude.	Height of centre.	Distance from the centre of the table.	Deviation of compass East.	Deviation of compass West.	Mean deviation.	Ratio of $\frac{\sin. 2\lambda}{\tan. \Delta}$
			inches.	inches.				
2° 30′ E. or W.	7° 27′	0° 0′	2·32	17·87	3° 0′	3° 0′	3° 0′	4·907
5 0	14 41	ditto	4·31	17·48	5 45	5 45	5 45	4·869
7 30	21 31	ditto	6·23	16·90	8 15	8 0	8 7½	4·780
10 0	27 51	ditto	7·92	16·17	10 0	10 0	10 0	4·686
12 30	33 35	ditto	9·39	15·36	11 0	11 0	11 0	4·741
15 0	38 45	ditto	10·62	14·53	11 40	12 5	11 52½	4·642
17 30	43 22	ditto	11·65	13·73	12 0	12 30	12 15	4·590
20 0	37 28	ditto	12·51	12·96	11 50	12 10	12 0	4·687
25 0	54 24	ditto	13·80	11·58	11 0	11 30	11 15	4·759
30 0	59 58	ditto	14·69	10·41	10 15	10 15	10 15	4·793
35 0	64 30	ditto	15·32	9·46	9 30	9 15	9 22½	4·707
40 0	68 18	ditto	15·77	8·69	8 30	8 30	8 30	4·595
50 0	74 21	ditto	16·32	7·56	6 20	6 20	6 20	4·681
60 0	79 6	ditto	16·66	6·82	4 30	4 30	4 30	4·719
70 0	83 4	ditto	16·84	6·35	3 0	2 45	2 57½	4·638
80 0	86 38	ditto	16·95	6·09	1 30	1 20	1 25	4·741
							Mean..	4·709

EXPERIMENTS.

26. *In the Circle S E, of which the Longitude is* 0°. *Ball* 288*lbs.*

RADIUS OF CIRCLE 20 INCHES.

Position of Compass.	Latitude.	Longitude.	Height of centre.	Distance from the centre of the table.	Deviation of compass East.	Deviation of compass West.	Mean deviation.	Ratio of sin. 2λ / tan. Δ
			inches.	inches.				
2° 30′ E. or W.	7° 27′	0° 0′	2·44	19·85	2° 15′	2° 15′	2° 15′	6·545
5 0	14 41	ditto	4·78	19·42	4 30	4 30	4 30	6·231
7 30	21 31	ditto	6·92	18·77	6 0	6 10	6 5	6·403
10 0	27 51	ditto	8·80	17·96	7 20	7 15	7 17½	6·456
12 30	33 35	ditto	10·43	17·06	8 10	8 15	8 12½	6·389
15 0	38 45	ditto	11·80	16·14	8 40	8 30	8 35	6·465
17 30	43 22	ditto	12·94	15·25	8 45	9 0	8 52½	6·394
20 0	47 28	ditto	13·89	14·39	8 40	8 45	8 42½	6·460
25 0	54 24	ditto	15·33	12·86	8 20	8 20	8 20	6·463
30 0	59 58	ditto	16·32	11·56	7 45	7 30	7 37½	6·473
35 0	64 30	ditto	17·02	10·51	6 45	7 0	6 52½	6·446
40 0	68 18	ditto	17·52	9·65	6 10	6 0	6 5	6·447
50 0	74 21	ditto	18·13	8·39	4 40	4 45	4 42½	6·308
60 0	79 6	ditto	18·51	7·57	3 20	3 20	3 20	6·376
70 0	83 4	ditto	18·71	7·05	2 10	2 10	2 10	6·338
80 0	86 38	ditto	18·83	6·76	1 0	1 0	1 0	6·717
							Mean..	6·432

27. The very near approximation of the numbers or ratios in the last column in each of the preceding tables towards an equality, cannot allow us for a moment to doubt that the law which I had stated in my former paper is actually the law of action between the iron and the compass; namely, *that the tangent of the angle of deviation, is proportional to the rectangle of the sine and cosine of the latitude, or to the sine of the double latitude, the*

the longitude being zero; that is to say, while the compass is carried round the globe in a great circle, passing from the east and west points of the horizon, perpendicular to the circle of no attraction, or equator Q Q′.

SECTION V.

DETERMINATION OF THE LAW OF ATTRACTION IN REFERENCE TO THE LONGITUDE.

28. Considering the law of action in the circle S E, completely established by the preceding experimental results, my next object was to ascertain the same for any other circle; that is, having found it for the latitude, I was next desirous of obtaining it likewise for the longitude; and hence, by combination, for any point on the globe.

In the account which I have given of my first course of experiments, I have stated no particulars of the few isolated observations I had registered with a view of deducing the law of action when the ball was moved out of the plane of the circle S E, although I had obtained a few results, which, combined with theoretical deductions relative to the resolution of forces, led me to assume that, where the latitude was the same, the tangent of the deviation would be found proportional to the cosine of the longitude, estimating the same from

the east and west points, that is to say, from the points where the horizon and equator intersect.

29. I felt however by no means the same confidence in this deduction as in that which had been experimentally determined for the latitude; for having no theoretical principle from which I could deduce this law of action, some uncertainty necessarily arose as to the proper method of resolving the oblique force. If I referred it by three rectangular co-ordinates to the planes of the circles Q Q′, S E, and S Q; it still remained doubtful whether the force which acted parallel to C S, had any influence in producing the deviation: if it had not, then the deviation ought to have followed the law which I had assumed; but if it had, the relations would become more complicated, although not difficult to compute; nor would there be any very considerable difference in the results, except in some particular points: I therefore undertook to compare all my deductions with both laws, and then to choose that which agreed best with my experimental observations. In this way I found that the law which I had before laid down, and which was much the most simple of the two, was also the most accurate, bringing the computed and observed deviations within very narrow limits: but there were still in this case some slight aberrations, rather greater perhaps

than could be properly attributed to the daily variation, or to errors of observations.

30. The method of submitting the above laws to the test of experiment was threefold; viz. I might move the compass over a great circle, of which the longitude should be constant; or over a small circle, in which the latitude was constant; or over one in which the latitude and longitude were both variable: for the sake of simplicity, in computing the requisite data, I chose the first and third of the above methods. According to the latter, I merely fixed my ball in the centre of the table, and carried the compass round in circles of different radii; and in the former, I made the following calculations.

31. Let H Z R (fig. 2) represent a sphere concentric with the iron ball at C, Q Q′ the equator, H R the horizon, N S the north and south poles, Z the zenith of the sphere, and S E the first meridian. Let S L P be a quadrant of any meridian, and let it be produced to meet H R in B, the longitude E P of the point P being supposed given, which will also be the longitude of any point L situated in that meridian. Imagine also a quadrant Z V to be drawn from Z, perpendicular to H R, and cutting the circle S P in L, also demit the perpendicular L M, which will fall in the line joining V and the centre C; so will C M (the cosine of the angle L C V to any proposed radius) represent the

distance of the point L, from the vertical axis of the ball, or from the centre of the table, and LM. (the sine of the same angle) the altitude of L above the plane HR. In order therefore to have the compass placed so as to coincide with the point L, the centre of the ball must be depressed below the plane of the table by a quantity equal to LM, at the same time that the compass must be approximated towards the centre of the table till its distance is equal to CM.

Hence if we assume any point P in the arc QQ′, and any point V in the arc EV, for the situation of the compass, we shall be able to compute the point L where these circles intersect each other; and hence the height LM, and the distance CM from the centre. Or, if instead of supposing the point P to be given, we suppose the point B to be assumed, then the longitude EP may be computed, and all the rest will be as above. In our case the latter is the most convenient, because we cannot with our division set off the point B on the horizontal plane to any fractional part of a degree.

32. Let us then assume EB=60°, and the angle QER=19° 30′ as before, then the triangle EPB being right angled at P, we shall have EB=60°, and the angle PEB=19° 30′, to find the base EP: this may be done by the formula,

$$\tan EP = \tan EB \cdot \cos PEB,^{*} \text{ or}$$
$$\tan EP = \tan 60^\circ \cdot \cos 19^\circ 30'.$$

Whence $EP = 58^\circ\ 31'$ the longitude of the arc SLP.

For the angle EBP we have

$$\tan EBP = \cos PEB \cdot \text{sec}\, EB, \text{ or}$$
$$\tan EBP = \cos 19^\circ 30' \cdot \text{sec}\, 60^\circ.$$

Whence $EBP = 79^\circ\ 57'$.

To find the perpendicular PB, we have

$$\sin PB = \sin EB = \sin PEB, \text{ or}$$
$$\sin PB = \sin 60^\circ \sin 19^\circ 30'.$$

Whence $PB = 16^\circ\ 48'$.

33. It now remains to assume different arcs for EV, and then from the preceding data to find the corresponding arcs BL, VL; from BL deducting BP, we shall have the latitude PL, of the point L, and the sine and cosine of LV, to any proposed radius will give the position of the ball and compass, as explained above. The formulæ for these determinations are as below; viz.

$$\begin{cases} \tan LB = \tan VB \cdot \text{sec}\, VBL \\ \tan LV = \sin VB \cdot \tan VBL. \end{cases}$$

* This and the three following equations, in the form of analogies, are as follow:

1. $\begin{cases} \text{rad} : \tan EB :: \cos PEB : \tan EP, \text{ or} \\ \text{rad} : \tan 60^\circ :: \cos 19^\circ 30' : \tan EP. \end{cases}$
2. $\text{rad} : \cos 19^\circ 30' :: \text{sec}\, 60^\circ : \tan EBP.$
3. $\text{rad} : \sin 60^\circ :: \sin 19^\circ 30' : \sin PB.$
4. $\begin{cases} \text{rad} : \tan VB :: \text{sec}\, VBL : \tan LB \text{ or} \\ \text{rad} : \sin VB :: \tan VBL : \tan LV. \end{cases}$

We have therefore only to assume for the arc V B, the successive arcs $2\frac{1}{2}°$, $5°$, $7\frac{1}{2}°$, $10°$, &c. and by introducing these into the above expressions, find the corresponding values of the arcs L B, and L V; the former of which, *minus* P B, will give the several latitudes, and the sines and cosines of the latter, the situation of the ball and compass.

In this manner all the numbers in the following tables have been computed, except those in the last column of each. The latter are deduced from the assumed law that the tangents of the angles of deviation are proportional to the rectangle of the sine of the double latitude and cosine of the longitude, which requires that the quotient $\frac{\sin 2\lambda \cdot \cos l}{\tan \Delta}$* should be a constant quantity: where λ is the latitude, l the longitude, and Δ the observed angle of deviation.

* That is, the product of the natural sine of the double latitude, and the natural cosine of the longitude, divided by the natural tangent of the angle of deviation ought to give a constant quotient.

34. *In the Circle S B, of which the Longitude is* 58° 31′. *Ball* 288*lbs.*

RADIUS OF CIRCLE 18 INCHES.

Position of Compass from B.	Latitude.	Longitude.	Height of centre.	Distance from the centre of the table.	Deviation of compass East.	Deviation of compass West.	Mean deviation	Ratio of sin. 2λ co / tan. Δ
			inches.	inches.				
2° 30′ E. or W.	2° 45′ S.	58° 31′	4·30	17·48	0° 30′	0° 30′	0° 30′	.....
5 0	9 50 N.	ditto	7·94	16·16	2 20	2 20	2 20	4·314
7 30	20 14	ditto	10·68	14·49	4 15	4 25	4 20	4·473
10 0	28 30	ditto	12·60	12·86	5 20	5 30	5 25	4·615
12 30	34 19	ditto	13·93	11·41	6 0	6 0	6 0	4·628
15 0	40 8	ditto	14·85	10·71	6 0	6 0	6 0	4·786
17 30	44 16	ditto	15·51	2·14	6 30	6 30	6 30	4·582
20 0	47 35	ditto	15·99	8·28	6 30	6 30	6 30	4·565
25 0	52 41	ditto	16·60	6·96	6 30	6 0	6 15	4·598
30 0	56 23	ditto	16·97	6·02	6 0	6 0	6 0	4·582
35 0	59 12	ditto	17·20	5·31	6 0	6 0	6 0	4·271
40 0	61 27	ditto	17·36	4·78	5 30	5 30	5 30	4·554
50 0	64 52	ditto	17·54	4·06	5 0	5 0	5 0	4·590
60 0	67 27	ditto	17·64	3·60	4 30	4 30	4 30	4·699

* This number is omitted in consequence of the latitude and deviation being of a contrary name to all that follow; the compass in this experiment being situate between P and B, fig. 2.

35. *In the Circle S B, of which the Longitude is* 58° 31′. *Ball* 288*lbs.*

RADIUS OF CIRCLE 20 INCHES.

Position of compass from B.	Latitude.	Longitude.	Height of centre.	Distance from the centre of the table.	Deviation of compass East.	Deviation of compass West.	Mean deviation	Ratio of sin. 2λ co / tan. Δ
			inches.	inches.				
2° 30′ E. or W.	2° 45′ S.	58° 31′	4·77	19·42	0° 30′	0° 45′	0° 37½′	.....
5 0	9 50 N.	ditto	8·82	17·95	1 30	1 30	1 30	6·719
7 30	20 14	ditto	11·86	16·10	3 10	3 10	3 10	6·135
10 0	28 30	ditto	14·00	14·28	4 15	4 0	4 7½	6·085
12 30	34 19	ditto	15·47	12·67	4 0	4 0	4 0	6·954
15 0	40 8	ditto	16·50	11·30	4 30	4 15	4 22½	6·741
17 30	44 16	ditto	17·23	10·15	4 30	4 45	4 37½	6·465
20 0	47 35	ditto	17·76	9·20	4 45	5 0	4 52½	6·109
25 0	52 41	ditto	18·44	7·73	5 0	4 45	4 52½	5·913
30 0	56 23	ditto	18·85	6·68	4 30	4 30	4 30	6·119
35 0	59 12	ditto	19·11	5·90	4 0	4 0	4 0	6·879
40 0	61 27	ditto	19·28	5·31	4 0	4 0	4 0	6·271
50 0	64 52	ditto	19·48	4·51	3 45	3 45	3 45	6·127
60 0	67 27	ditto	19·59	4·00	3 30	3 30	3 30	6·048

36. The numbers in the last columns of the two preceding tables, do not, as I have before observed, approximate so nearly to uniformity as those from which I deduced the law of the latitude; but, on the other hand, it ought to be remarked, that the deviations are considerably smaller than in that case; and, consequently any trifling error of observation produces a more sensible effect, while the compass itself is brought much nearer to the centre of the table, where a small error in adjusting it is more easily made, and gives rise to a greater discrepancy. Upon the whole, therefore, I conceive that I am justified in saying, that the law as regards the longitude has been determined, at least approximatively, and that it is such, *that while the latitude is constant, the tangent of the deviation is proportional to the cosine of the longitude.*

The proper correction of this first approximative law will be found in Part II.

37. In the preceding experiments the latitude varied, while the longitude remained constant; I therefore thought it desirable to pass the compass round the ball in a circle, in which both these quantities should change; and, as the most convenient, I chose the horizontal circle H R, bringing down the ball, in the first place, exactly into its central position; viz. so that its centre coincided with the horizontal plane passing through the pivot of the needle; in which case the errors at east and

west became zero. I had then only to assume the arc E B successively equal to 5°, 10°, 15°, &c. and to compute the corresponding arcs of latitude P B, and of longitude E P; then adopting the mean numbers given in my experiments for the latitude, I computed the deviation by means of the formula

$$\tan \Delta = \frac{\sin 2\lambda \cos l^*}{A}$$

where A is the mean tabular number corresponding to any given distance. The following tables exhibit the comparison between these computed and observed deviations.

* That is I divided the product of the natural sine of the double latitude, and natural cosine of the longitude, by the constant numbers determined in Table 2 and 3, page 26, 27. viz. by 2·737 and 4·709; which ought, obviously, according to the preceding law, to give the natural tangents of the angles of deviation in every position.

It may be proper to observe, that in the former edition these numbers had been found 2·726 and 4·717, but it has not been thought necessary to recompute the column of deviations.

EXPERIMENTS.

38. *In the Circle H R. Ball* 288*lbs.*

RADIUS 15 INCHES.

Position of compass from E. or W.	Latitude.	Longitude.	Deviation East.	Deviation West	Mean of observed deviation	Computed deviation	Errors.
5° 0′	1° 40′	4° 24′	1° 15′	1° 20′	1° 17½′	1° 14′	+ 3½′
10 0	3 19	9 26	2 25	2 20	2 22½	2 24	— 1½
15 0	4 57	14 10	3 0	3 30	3 15	3 33	— 18
20 0	6 33	18 56	4 15	4 30	4 22½	4 33	— 10½
25 0	8 6	23 43	5 0	5 20	5 10	5 21	— 11
30 0	9 36	28 33	5 45	6 0	5 52½	6 7	— 14½
35 0	11 2	33 25	6 15	6 30	6 22½	6 38	— 15½
40 0	12 33	38 20	6 45	7 0	6 52½	7 2	— 9½
45 0	13 38	43 18	6 45	7 0	6 52½	7 3	— 10½
50 0	14 48	48 19	6 45	7 0	6 52½	6 58	— 5½
55 0	15 52	53 24	6 10	6 30	6 20	6 39	— 19
60 0	16 48	58 30	5 45	6 15	6 0	6 7	— 7
65 0	17 36	63 41	5 10	5 30	5 20	5 25	— 5
70 0	18 16	68 53	4 20	4 30	4 25	4 33	— 8
75 0	18 49	74 8	3 45	4 10	3 57½	4 20	— 22½
80 0	19 11	79 24	2 24	2 30	2 27	2 24	+ 3
85 0	19 25	84 42	1 14	1 20	1 17	1 14	+ 3
North	19 30	90 0	0 0	0 0	0 0	0 0	0

Note. The observed deviations, as given in the above and in the following table, are the means of three distinct series of observations.

EXPERIMENTS.

39. *In the Circle H R. Ball 288lbs.*

RADIUS 18 INCHES.

Position of Compass from E. or W.	Latitude.	Longitude.	Deviation East.	Deviation West	Mean observed deviation	Computed deviation	Errors.
5° 0′	1° 40′	4° 24′	0° 45′	0° 45′	0° 45′	0° 43′	+ 2′
10 0	3 19	9 26	1 20	1 25	1 22½	1 24	— 1½
15 0	4 57	14 10	2 10	2 0	2 5	2 3	+ 2
20 0	6 33	18 56	2 40	2 40	2 40	2 41	— 1
25 0	8 6	23 43	3 10	3 10	3 10	3 9	+ 1
30 0	9 36	28 33	3 30	3 40	3 35	3 33	+ 2
35 0	11 2	33 25	4 0	4 0	4 0	3 52	+ 8
40 0	12 33	38 20	4 0	4 0	4 0	3 59	+ 1
45 0	13 38	43 18	4 0	4 0	4 0	4 2	— 2
50 0	14 48	48 19	4 0	4 0	4 0	4 2	— 2
55 0	15 52	53 24	3 50	3 50	3 50	3 48	+ 2
60 0	16 48	58 30	3 30	3 40	3 35	3 33	+ 2
65 0	17 36	63 41	3 10	3 0	3 5	3 6	— 1
70 0	18 16	68 53	2 40	2 40	2 40	2 39	+ 1
75 0	18 49	74 8	2 0	2 0	2 0	2 2	— 2
80 0	19 11	79 24	1 30	1 30	1 30	1 24	+ 6
85 0	19 25	84 22	0 40	0 45	1 42½	0 42	+ 0½
North	19 30	90 0	0 0	0 0	0 0	0 0	0

40. In the above experiments the law which I have suggested has been put to a severe test, in consequence of the change, both in latitude and longitude, and particularly the latter; which varies from zero to 90°; and the near agreement between the observed and computed results, more especially in the second table, seems to leave no reasonable doubt of the close approximation of our preceding

deduction; viz. that while the mass and distance is the same, "*the tangent of the deviation is proportional to the rectangle of the cosine of the longitude, and the sine of the double latitude.*" We have already referred for the correct law to Part II.

41. The nature and properties of the plane of no attraction, and the law for latitude and longitude of position, will be better understood by some readers, by referring to (fig. 3.) In which O is supposed to represent an iron ball, and A A A a sphere circumscribing it, and within which its influence is active; S Q N Q′ being in the magnetic meridian.

The line N S in the plane S E N W denotes the natural direction of the dipping needle, in these latitudes where its inclination to the horizon is about 70°½. Now conceiving Q E Q′ W to represent a circle or plane, passing through the centre of the ball, and perpendicular to the axis N S, it will be the *plane of no attraction*, which, as we have seen, has this remarkable property, that, if lines be drawn in it (as for example the lines O C, O C′, O C″, &c.), and a compass be placed any where in those lines, or in short in any point of the plane Q E Q′ W, it will be uninfluenced by the iron ball, and it will preserve its natural magnetic direction.

But as soon as the compass is removed out of this plane, the needle is found to deviate from its original bearing; its south end being drawn to-

wards the ball, when the needle is below the plane, and its north end when it is above, and in every case the deviation follows the determinate law above indicated.

Let us for example, conceive any two other planes passing through the centre of the ball, and each perpendicular to QEQ′W, of which let MOSL, M′OSL′ represent quadrants; then supposing a compass placed in each of these planes, at equal distances from the centre, as at L and L′, we shall have ML, M′L′, for the latitude, and EM, EM′, for their longitude of position; and

The tangent of the deviation of the compass at L
Is to the tangent of deviation of the compass at L′
As the rectangle of the sine of 2ML × cosine of EM
To the rectangle of the sine of 2. M′L′ × cosine of EM′
E being the east point of the horizon.*

SECTION VI.

DETERMINATION OF THE LAW OF ATTRACTION AS REGARDS THE DISTANCE.

42. Having in my experiments respecting the latitude obtained four mean results, corresponding to the four distances, 12, 15, 18, and 20 inches,

* I am indebted to Mr. A. Ainger for the above very simple and perspicuous illustration and drawing.

let us examine whether any law obtains between those mean ratios and any power or powers of the distances. These ratios and distances are as below.

Distance.	Ratios.
12 inches.....	1·365
15.....	2·737
18.....	4·709
20.....	6·432.

In order to investigate this question, let m denote any indeterminate index, and let us assume that

$$12^m : 15^m :: 1365 : 2737$$
$$12^m : 18^m :: 1365 : 4709$$
$$12^m : 20^m :: 1365 : 6432$$
$$15^m : 18^m :: 2737 : 4709$$
$$15^m : 20^m :: 2737 : 6432$$
$$18^m : 20^m :: 4709 : 6432.$$

If, in all these proportions, we find m equal, or nearly equal, to one determinate number, we may conclude that such value of m will express the power of the distance, which is proportional to the inverse ratio of the tangents of the deviations, the position of the ball and compass remaining the same in respect to latitude and longitude.

Now the preceding proportions give

$$1 \ldots\ldots m = \frac{\log 2737 - \log 1365}{\log 5 - \log 4} = 3{\cdot}118$$

$$2 \ldots\ldots m = \frac{\log 4709 - \log 1365}{\log 3 - \log 2} = 3{\cdot}054$$

$$3 \ldots\ldots m = \frac{\log 6432 - \log 1365}{\log 5 - \log 3} = 3{\cdot}035$$

$$4 \ldots\ldots m = \frac{\log 4709 - \log 2737}{\log 6 - \log 5} = 2{\cdot}976$$

$$5 \ldots\ldots m = \frac{\log 6432 - \log 2737}{\log 4 - \log 3} = 2{\cdot}970$$

$$6 \ldots\ldots m = \frac{\log 6432 - \log 4709}{\log 10 - \log 9} = 2{\cdot}959$$

Mean . . 3·019.

The approximation of all these ratios towards the integer 3, seems to indicate that the tangents of the deviations vary reciprocally as the cubes of the distances, the position, with regard to latitude and longitude, remaining the same; but as this law is not exactly uniform in the above results, let us examine and determine the least change that can be made in the numbers, as drawn from our experiments, in order that a complete coincidence may be obtained.

This will be best done by a particular application of the method of *minimum squares*, as follows. Let $1{\cdot}365 = a$, $2{\cdot}737 = a'$, $4{\cdot}709 = a''$, and $6{\cdot}432 = a'''$.

Now, if these numbers had the exact ratio of the cubes of the respective distances 12, 15, 18, 20, they would, by the supposition, require no correction; but this not being the case, let us endeavour to find the four quantities x, x', x'', x''', which added respectively to a, a', a'', a''', shall make,

$$a + x,\ a' + x',\ a'' + x'',\ a''' + x''',$$

proportional to 12^3, 15^3, 18^3, 20^3,

and such also that

$$x^2 + x'^2 + x''^2, + x'''^2 = a\ minimum.$$

That is, we must have

$$\frac{a + x}{a' + x'} = \left(\frac{12}{15}\right)^3 = \left(\frac{4}{5}\right)^3 = b$$

$$\frac{a + x}{a'' + x''} = \left(\frac{12}{18}\right)^3 = \left(\frac{2}{3}\right)^3 = b'$$

$$\frac{a + x}{a''' + x'''} = \left(\frac{12}{20}\right)^3 = \left(\frac{3}{5}\right)^3 = b''$$

and $x^2 + x'^2 + x''^2 + x'''^2 = a\ min.$

The first three equations give

$$a + x = b\ a' + b\ x'$$

$$a + x = b'\ a'' + b'\ x''$$

$$a + x = b''\ a''' + b''\ x'''$$

$$\text{or, } x' = \frac{a - b\,a' + x}{b} = \frac{x + c}{b}$$

$$x'' = \frac{a - b'\,a'' + x}{b'} = \frac{x + c'}{b'}$$

$$x''' = \frac{a - b''\,a''' + x}{b''} = \frac{x + c''}{b''};$$

consequently,

$$\left(\frac{x + c}{b}\right)^2 + \left(\frac{x + c'}{b'}\right)^2 + \left(\frac{x + c''}{b''}\right)^2 + x^2 = a\ min.$$

This expression, when reduced, gives

$$x = -\left(\frac{c\,b'^2\,b''^2 + c'\,b^2\,b''^2 + c''\,b^2\,b'^2}{b^2\,b'^2 + b^2\,b'^2 + b'^2\,b''^2 + b^2\,b'^2\,b''^2}\right)$$

Whence we find in numbers $x = \cdot 024$; and thus the preceding ratios, when corrected, become

$$\left.\begin{array}{l} x + a = 1{\cdot}389 \\ x' + a' = 2{\cdot}712 \\ x'' + a'' = 4{\cdot}687 \\ x''' + a''' = 6{\cdot}430 \end{array}\right\} \text{the corrections being} \left\{\begin{array}{l} {\cdot}024 \\ {\cdot}025 \\ {\cdot}022 \\ {\cdot}002 \end{array}\right.$$

43. These numbers have exactly the inverse ratio of the cubes of the distances, and either of them will give for the number corresponding to the unit of distance, A = ·00080382. We have therefore the following formula for computing the deviation for any distance, the latitude and longitude being given, and the mass remaining the same, viz.

$$\tan \Delta = \frac{\sin 2\lambda . \cos l.}{{\cdot}00080382\, d^3}$$

where λ is the latitude, l the longitude, and d the distances in inches.

44. As the above constant coefficient is obtained from our corrected numbers, and as it might appear doubtful how far these corrections would effect the uniformity of the law as determined from the preceding experiments, I undertook to compute and observe the deviations in four different positions of the compass for the several circles whose radii are 12, 14, 16, 18, 20, 22, and 24 inches selecting for those positions of the compass the divisions 15°, 17½°, 20°, 25°; from the east or west, these being the points where the deviations are the greatest, in order that any error might render itself the more obvious. The following are the results of these observations with the corresponding computed deviations:

EXPERIMENTS.

45. *In the Circle S E to several radii. Ball 288lbs.*

sition Com- s from or W.	Latitude.	Longi- tude.	Distance.	Computed deviation, $\tan\Delta = \frac{\sin 2\lambda . \cos l.}{\cdot 00080382^{*} d^{3}}$	Mean of observed deviation	Errors
			inches.			
15°	38° 45′	0° 0′	12	35° 18′	35° 37′	+19
17½	43 22	ditto	12	35 54	36 15	+21
20	47 28	ditto	12	35 50	35 50	0
25	54 24	ditto	12	34 28	34 30	+ 2
15	38 45	ditto	14	24 2	24 0	— 2
17½	43 22	ditto	14	24 31	24 30	— 1
20	47 28	ditto	14	24 28	24 30	+ 2
25	54 24	ditto	14	23 23	23 30	+ 7
15	38 45	ditto	16	16 37	16 30	— 7
17½	43 22	ditto	16	16 59	17 0	+ 1
20	47 28	ditto	16	16 56	17 0	+ 4
25	54 24	ditto	16	16 9	16 0	— 9
15	38 45	ditto	18	11 51	11 52	+ 1
17½	43 22	ditto	18	12 7	12 15	+ 8
20	47 28	ditto	18	12 5	12 0	— 5
25	54 24	ditto	18	11 30	11 15	—15
15	38 45	ditto	20	8 42	8 35	— 7
17½	43 22	ditto	20	8 53	8 52	— 1
20	47 28	ditto	20	8 52	8 42	—10
25	54 24	ditto	20	8 26	8 20	— 6
15	38 45	ditto	22	6 34	6 30	— 4
17½	43 22	ditto	22	6 42	6 40	— 2
20	47 28	ditto	22	6 41	6 30	—11
25	54 24	ditto	22	6 22	6 25	+ 3
15	38 45	ditto	24	5 4	5 0	— 4
17½	43 22	ditto	24	5 11	5 15	+ 4
20	47 28	ditto	24	5 10	5 10	0
25	54 24	ditto	24	4 55	5 0	+ 5

* This number in the former edition was ·000798125, it has not been thought necessary to recompute this column of deviations; the difference being necessarily very inconsiderable.

46. The remarkable coincidence, or at least th close approximation, between the observed an computed results in these experiments, can leav no doubt that the law of attraction as respects th distance is, *that the tangents of the angles o deviation are reciprocally proportional to th cubes of the distances.*

Since the magnetic force varies inversely as th square of the distance, and the tangent of deviatio inversely as the cube of the same; it follows tha the square of the tangent of deviation is directl as the cube of the force; or that *the tangent o deviation varies directly as the* $\frac{3}{2}$ *power of th force.*

SECTION VII.

DETERMINATION OF THE LAW OF ATTRACTION I REFERENCE TO THE MASS, &c.

47. It seemed highly probable that the effect, o power of attraction, would be found to follo the direct law of the mass; but in order to procee as I had hitherto done, rather on the ground o experiment than hypothesis, I procured a soli 10 inch ball, such as is employed in proving th 10 inch mortars, the weight being 128lbs. viz. jus $\frac{1}{8}$ths of the weight of the 13 inch ball; and wit this I repeated the series of experiments given a pages 23 and 24. The following are the results o these operations:

EXPERIMENTS.

48. *In the Circle S E, with Balls of* 288*lbs. and* 128*lbs.*

RADII 15 INCHES AND 12 INCHES.

Position of Compass.	Distance 12 inches.		Ratio of Tangents.	Distance 15 inches.		Ratio of Tangents.
	Deviation Ball 288lbs.	Deviation Ball 128lbs.		Deviation Ball 288lbs.	Deviation Ball 128lbs.	
2° 30′	10° 7½′	4° 0′	2·552	5° 7½′	2° 15′	2·279
5 0	19 37½	8 30	2·385	10 7½	4 30	2·267
7 30	26 30	11 45	2·397	14 7½	6 30	2·207
10 0	31 45	15 15	2·269	16 52½	7 45	2·227
12 30	34 7½	17 0	2·216	18 37½	8 30	2·254
15 0	35 37½	17 30	2·272	19 45	9 0	2·267
17 30	36 15	18 15	2·353	20 15	9 15	2·265
20 0	36 15	18 15	2·353	20 7½	9 15	2·250
25 0	34 30	17 0	2·248	19 15	8 45	2·269
30 0	32 22½	15 45	2·246	17 37½	8 0	2·259
35 0	29 52½	14 15	2·261	16 0	7 15	2·254
40 0	26 45	12 0	2·386	14 5	6 15	2.319
50 0	21 0	9 45	2·238	10 45	4 45	2·285
60 0	15 15	7 0	2·222	7 30	3 20	2·261
70 0	10 0	4 0	2·521	5 0	2 15	2·227
80 0	4 52½	2 15	2·172	2 30	.....	.....

49. The mean of the first ratios is 2·318, and of the second ratios 2·259; and the mean of the two 2·288. Now the ratio of the masses, or of the cubes of the diameter, is 2·25, whence then we may conclude *that the tangents of the deviations are proportional to the cubes of the diameters,* all other things being the same.

50. The cube of the diameters being proportional to the masses, the obvious conclusion seemed to be, that the tangents of the deviation were also proportional to the masses; and such, in fact, was

the conclusion I had drawn, when I fortunately made trial of a 10 inch shell, whose weight was 96lbs. or just $\frac{3}{4}$ths of that of the last solid ball of the same dimensions; and I was not a little surprised to find, that I could observe no difference whatever between these results and the former. I then determined on a regular course of experiments with the shell, at the same distances, &c. as I had adopted with the ball; and having completed them, I found, on a comparison of the results that they tallied with each other throughout. In fact, it appeared *that the power of attraction resided wholly on the surface, and was independent of the mass.*

51. Being, however, unwilling to leave any thing doubtful respecting a result which appeared so extremely novel and unexpected, I tried two other 10 inch shells, lest there should have been any thing peculiar in the one above referred to: I then employed other shells of different diameters and thicknesses, the whole of which still indicated the same law;* viz. that, *the tangents of the deviation are proportional to the cubes of the diameters, or to the $\frac{3}{2}$ power of the surfaces, whatever may be the weight and thickness.* Here

* I shall not detain the reader by giving the detail of these experiments, as it would consist merely of a repetition of the same numbers as those stated in the preceding tables.

again, the magnetic force being as the surface, and the tangent of deviation as the $\frac{3}{2}$ power of the surface, it follows also, that the square of the tangent of deviation varies directly as the cube of the force, or the tangent of deviation directly as the $\frac{3}{2}$ power of the force; which is the same conclusion that we have drawn from (art. 46).

52. This law, however, has its limits; for having procured a 10 inch shell of tin, and another of iron, the weight of the former being 2lbs. 11oz., or 43oz.; and of the latter, 2lbs. 13oz., or 45oz.; I found the power not so great as in the solid ball of iron, although the approximation was very near, considering the great disparity in the weights; the iron shell producing deviations, which were to those of the solid ball as 2 to 3, nearly. Now the thickness of the iron being here at a medium about $\frac{1}{10}$th of an inch, the conclusion which we may draw from this fact appears to be, that the magnetic fluid requires a certain thickness of metal, exceeding $\frac{1}{10}$th of an inch, in order effectually to develope itself, and to act with its maximum of power.

The tin shell produced a similar effect in some positions, and a greater in others, arising obviously from its having imbibed (probably in the operation of *raising it*, as is termed by workmen, to the globular form), a partial degree of magnetism, an effect from which the iron shell was not wholly

free; and to which we may probably, in part, attribute the discrepancy alluded to above. This is, indeed, rendered the more probable, from my having, in the first instance, employed a tin vessel of a globular form, and of about the same dimensions as my 10 inch shell, but which had not been strained in the same degree by the workmanship, from which results were obtained fully equal to those of the solid ball.

Upon the whole, therefore, I may venture to conclude, (although further experiments are necessary, fully to confirm the fact), that the magnetic power resides wholly on the surface of iron bodies, but that a certain thickness of metal is necessary for its complete developement; a remarkable instance of the analogy which subsists between the magnetic and electric fluids.*

53. Since the publication of the first edition of this work, Captain Kater has made a short set of experiments, with a view of verifying the above deduction.

For this purpose, he had three cylinders made of

* The analogy between these two sciences, already known, had led Mr. Charles Bonnycastle to suggest to me, some time before, that it would be desirable to ascertain whether they resembled each other in this respect also; but being myself decidedly of opinion that the power resided in the mass, I made no trial on the subject, till led to it accidentally, as above stated.

soft iron, about $2\frac{1}{2}$ inches in diameter, and nearly the same in height. "One of the cylinders was of sheet iron, less than $\frac{1}{20}$th of an inch in thickness; the second of that kind called chest plate, 0·185 inch thick; and the third was of solid wrought iron. The first weighed 2760 grains, the second 9376, and the solid cylinder 22929 grains. Previous to the experiments, they were all made red hot to destroy any accidental magnetism."

The compass employed was of a very delicate construction, and the cylinder was so placed that its centre was in the direction of a tangent to the zero of the compass, and at the distance of 4·85 inches from the southern extremity of the needle. The position of the cylinder was varied six times, and the following were the deviations of the needle:

Sheet iron Cylinder.	Chest plate Cylinder.	Solid Cylinder.
2° 1·5′	2° 50′	2° 55′
2 1·5	3 4	3 15
2 4·5	3 20	2 57
2 5	3 45	2 50
2 5	3 10	2 55
2 10	3 30	2 30
Mean . . 2 16	3 16	2 54

Suspecting an error in the experiment with the solid cylinder, from an accident which occurred, Captain Kater repeated the whole with the utmost

attention. The position of each cylinder was now varied eight times, and the results were as follows:

Sheet Iron Cylinder.		Chest plate Cylinder.		Solid Cylinder.	
2°	3′	2°	55′	3°	15′
2	22	2	50	3	12
2	32	3	20	3	15
2	20	3	40	3	0
1	50	3	40	3	15
2	45	3	28	2	50
2	45	3	10	2	45
1	55	3	5	2	58
Mean.. 2	19	3	16	3	4

"The surfaces of the cylinders determined by very careful measurement, were, the sheet iron 28·54 inches; the chest plate 30·77; and the solid cylinder 28·94 inches."

"Reducing the deviations to the same extent of surface; viz. to that of the solid cylinder, they become respectively 141, 184 and 184 minutes." *Philosophical Transactions for* 1821.

SECTION VIII.

SUPPLEMENTARY EXPERIMENTS, RELATIVE TO THE QUANTITY OF ATTRACTION AS REGARDS THE THICKNESS OF METAL.

54. Having received permission, from my Lords Commissioners of the Admiralty, to avail

myself of any facilities which His Majesty's Dock Yard, at Woolwich, might afford in pursuing my inquiries, I ordered plates of iron, a foot square, to be formed of the various kinds that could be procured from the stores; and I thus obtained the specimens as stated in the following page, varying from 1-50th of an inch to one fifth of an inch in thickness.

These being prepared, I placed my compass so that it read very accurately zero at the north and south, and in a situation where I could set up in succession the several plates at ten inches distance from the pivot of the needle, and exactly parallel to its direction; keeping the centre of the plate three inches above the point or pivot of suspension. Each plate was placed in four successive positions, viz. with each of its four sides downwards, the deviation taken in these respective positions, and then a mean of the four, as given in the table; these precautions being rendered necessary in consequence of the partial magnetism which the plates all exhibited in a greater or less degree. The following are the results of these experiments:

55. *Experiments on Plates of Iron, a foot square, of various Thicknesses.*

No.	Weight in ounces. avoirdupoise	Thickness in decimals of an inch.	Deviations.	Mean deviations.	Denomination of the iron in commerce.
1	14·5	·0223	2° 15′ 2 30 2 0 1 30	2° 4′	Tin plate.
2	20·0	·0308	2 0 3 0 2 45 2 0	2 26	Number 22 plate iron
3	20·5	·0315	3 0 2 0 0 30 2 0	1 52½	Number 22 ditto.
4	23·5	·0357	3 0 3 30 2 30 2 0	2 45	Number 20 ditto.
5	31·0	·0477	4 30 5 0 4 30 4 0	4 30	Number 18 ditto.
6	42·0	·0646	5 30 5 0 5 0 5 30	5 15	Number 16 ditto.
7	48·0	·0738	4 0 4 0 4 0 4 30	4 7½	Number 15 ditto.
8	48·0	·0738	3 30 4 15 4 30 4 0	4 3¾	Ditto ditto.
9	90·0	·1384	6 30 6 30 5 45 6 15	6 15	Chest iron.
10	122	·1877	5 30 5 45 5 0 5 0	5 18	Common plate.
11	122	·1877	5 0 5 30 4 0 4 0	4 37½	Ditto.
12	130	·2000	5 0 5 15 4 0 4 15	4 47	Ditto.

56. If we examine the mean results given by the preceding experiments, it will be obvious, that beyond a certain thickness (which appears to be necessary for the entire developement of the magnetic fluid) the quantity of deviation no longer depends upon the mass; being greater in some cases where the mass is less, and less where it is greater. It appears also that different plates, presenting the same surface, and of the same thickness, give different results; contrary to what was found to be the case in balls of iron of the same diameter. These irregularities must, I believe, be attributed principally to the partial magnetism noticed above, although a part of them may arise from a greater purity of the metal in one plate than in another: but, however this may be, it is evident that the attracting power is independent of the mass, provided only the thickness of the plate exceed a certain quantity (probably 1-20th of an inch), and that it resides principally in the surface, which is the same conclusion as I have already drawn from the experiments detailed in Section VII.

57. The partial magnetism in most of the above plates, and which seems to be common to all small masses of iron, disappears entirely in larger masses: at least I was never able to detect it, in any sensible degree, in the balls which I employed in my experiments; and to this circumstance I must attribute

my success in the developement of the laws of action; for these never could have been rendered obvious, had there been any inequality of deviation arising from the unequal distribution of the attracting power.

Another point of difference between these series of plates, and the balls employed in the former experiments, is, that in one case the iron is milled and laminated, and in the other cast and granulous; which latter process and constitution may be much more favourable than the former for producing an uniform distribution of the magnetic fluid: some experiments directed to this inquiry, are given in a following section.

SECTION IX.

SUMMARY OF THE PRECEDING DEDUCTIONS.

58. There exists a plane of *no attraction* in every ball or shell of iron, which plane, in this latitude, inclines from north to south, and forms, with the horizon, an angle equal to the complement of the dip.

59. Considering any circle in this plane as an equator to an ideal sphere concentric with the ball or shell, and imagining circles of latitude and longitude to be drawn thereon, the first meridian being

supposed to pass through the east and west points, we shall have (while the diameter of the ball and the distance are the same,) the tangent of the angle of deviation proportional to the rectangle of the sine of the double latitude, and the cosine of the longitude* of the place of the compass as referred to the above sphere.

60. Instead of conceiving the imaginary sphere to surround the ball, we may imagine a similar sphere concentric with the pivot of the needle, then it is obvious that the centre of the ball will have the same relative position on the latter sphere as the pivot of the compass has with respect to the former, so that the reference may be made to either at pleasure; but when the mass of iron is irregular, it will then be preferable to refer the common centre of attraction of the iron to the imaginary sphere circumscribing the compass.

61. The distance only being variable, the tangent of deviation is reciprocally proportional to the cube of the distance.

62. All other things being the same, the tangents of deviation are proportional to the cubes of the diameters of the balls or shells, whatever may be their masses, provided only that the thickness exceed a certain quantity.

* It will be seen in a subsequent section, that this law, with respect to the cosine of the longitude, requires, in certain cases, a slight modification.

63. All these laws may be expressed by the general formula

$$\tan \Delta = \frac{D^3}{A\,d^3} (\sin 2\lambda \cos l) \text{ or}$$

$$\tan \Delta = \frac{r^3}{A\,d^3} (\sin 2\lambda \cos l) \text{ or}$$

where Δ is the angle of deviation, λ the latitude, and l the longitude on the ideal sphere, D the diameter and r the radius of the ball or shell, d the distance of the centre of the ball from the pivot of the needle, and A a constant coefficient, to be determined by experiment.

64. Since the force has been shown to vary with the surface or square of the diameter, independently of the mass, while the tangents of deviation are as the cubes of the diameters, it follows that the squares of the tangents of deviation are directly proportional to the cubes of the forces.

65. The same inference may also be drawn from the law of the distances; by assuming, that the force varies inversely as the square of the distance. For then, the tangents being inversely as the cubes of the distances, while the force varies as the squares of the same, it follows, as above, that the squares of the tangents of deviation are directly as the cubes of the forces.

66. It should be observed, that the coefficient A, deduced in the preceding experiments, has been obtained by observations on one compass only, and

that, when other compasses have been employed, a different coefficient has been obtained. The discrepance, however, in this respect, I have found extremely small, as far as my observations have extended, notwithstanding the needles I have made use of have varied very essentially from each other. The instrument belonging to Mr. Arrowsmith, for instance, is very unlike, in form, length, weight, and intensity, those of my other three compasses: and, besides these, I made trial of an excellent small pocket compass, the needle of which was only 1½ inch in length, and its weight not above 1-30th part of that of the large needle above referred to; yet neither I, nor Dr. Gregory, of whom I borrowed it, could perceive the least difference in the deviation of the two when placed in the same situations. May we not then attribute the small difference that was observed, rather to some defect in the suspension, than to any other cause?

Some of the instruments also gave different values for A, accordingly as the upper or under half of the ball was opposed to them; which I conceive must have arisen from a similar cause.

67. In conclusion I must observe, that in every instrument there is a limit within which the above laws cease to obtain. It is well known that, within certain distances, iron will attract either end of the needle that is presented to it; which is commonly

explained by supposing that the iron has then been actually magnetized by the needle. What the distances are at which this effect begins to be produced, whether it be different with different needles, and with great and small masses of iron, &c. are questions to which, for want of leisure, I have not attended, although they appear to be deserving of a more attentive consideration than has hitherto been bestowed upon them. That such is the fact, however, is well known; and when the compass is brought so near the iron as to be thus circumstanced, the laws which I have laid down must necessarily fail. I have, in my experiments, always worked at such a distance from the mass of iron as to be far beyond the limit alluded to: it would, however, be an interesting inquiry to ascertain precisely what those limits are, and the laws and circumstances on which they depend; but this I must leave, hoping that some one, with more leisure than myself, may be induced, from what is above stated, to undertake the investigation, which I have little doubt would produce results that would amply compensate for the time and attention thus bestowed.

SECTION X.

EXPERIMENTS ON A TWENTY-FOUR POUNDER GUN, ON A TRAVERSING PLATFORM, IN THE ROYAL MILITARY REPOSITORY, WOOLWICH.

68. The law of action, as it obtains between a magnetic needle and *regular* masses of iron, being established, either exactly or approximatively, by the foregoing experiments, my next object was to ascertain whether the same law had place in *irregular* masses; which was by no means obvious, particularly in admitting the present received doctrine of magnetic attractions, which gives to every individual part of the entire mass two distinct poles. In the manner in which I had viewed the subject, by referring the entire action to one common centre of attraction, the matter was less doubtful. It seemed, however, at all events highly desirable to submit the accuracy and generality of my preceding deductions to some such test as that which forms the subject of this section. I therefore addressed a letter to Sir William Congreve, requesting permission to pursue my inquiries in the Royal Military Repository at Woolwich, which place offered many advantages and facilities of operation; and I could not but feel myself highly flattered and obliged by the polite and handsome manner in which that

gentleman answered my communication, and by the readiness with which he granted my request.

I immediately selected for my purpose an iron *twenty-four pounder*, mounted on a platform which admitted of its being traversed through an entire circumference; the trucks at the bottom running over a circle ten feet six inches in diameter.

69. After ascertaining the magnetic north point of this circle, I had it divided into 32 equal parts, corresponding to the points of the compass; whereby I was enabled to carry the gun round, and set it in any required position. I then had a piece of wood formed to fit the bore of the gun inside, and projecting beyond the muzzle above four feet. This projecting part was semi-cylindrical, the flat side exactly bisecting the bore horizontally; and on this piece the compass was placed, at different distances, while the gun was traversed, point by point, from north to north again, as stated in the following tabulated results.

70. As my preceding experiments had shown that much depended upon the angle formed between the compass and the plane of the *circle of no attraction*, and being unable, from the irregular nature of the mass, to compute the place of the centre of attraction, it was necessary to determine the situation of the above plane experimentally. In order to effect this, the compass was placed on the projecting piece above described, at 18 inches

distance from the muzzle, and the gun rendered, by means of the breech screw, truly horizontal; being now pointed north and south, the needle also attained its true direction; which was a proof that the compass was equally acted upon by the iron of the gun and carriage, to the right and left of the line to which it was adjusted. But on bringing the muzzle east or west, the needle was found to deviate about 15° from its true direction; the deviation being east with the muzzle at the west, and west with the muzzle at the east; which indicated that the centre of attraction of the mass was below the horizontal plane passing through the pivot of the needle: the gun was therefore gradually depressed till the needle pointed due north and south. In performing this operation it was observed that every degree of depression caused a change of about 4° in the direction of the compass; and having at length brought the needle due north and south, the angle of depression was found to be 3° 58′: at greater distances this angle was a few minutes less, as ought obviously to be the case; because, as the compass was carried farther from the muzzle, it descended below the plane of the above circle. This striking confirmation of the existence of the plane of no attraction in the most irregular masses of iron was highly gratifying to me, and equally so to those who witnessed the experiments.

71. Having thus found the position of the gun

such, that the compass read correctly when the piece was placed at the four principal points, north, east, west, and south, I caused it to be traversed, as above stated, point by point, from north to north again; and the following are the observed deviations.

EXPERIMENTS

72. *On an Iron Twenty-four Pounder Cannon. Length, 9 feet 6 inches; t Distance of Trunnions from the Muzzle, 5 feet 6 inches; Weight of the Ma 58* cwt.*

Distance of Compass from the Muzzle, 2 feet 6 inches; Depression, 3° 4

Position of the Trail of the Carriage.	Deviation of the Needle.	Position of the Trail.	Deviation of the Needle.	Position of the Trail.	Deviation of the Needle.	Position of the Trail.	Deviation of the Needle.	Me Co put devia
East.	0° 0′ E.	West.	0° 0′ W.	East.	0° 0′ W.	West.	0° 0′ E.	0°
11¼° N.	4 15	11¼° N.	4 15	11¼° S.	3 45	11¼° S.	4 0	3
22½ N.	7 30	22½ N.	7 30	22½ S.	6 30	22½ S.	7 30	6
33¾ N.	9 0	33¾ N.	9 15	33¾ S.	9 15	33¾ S.	9 0	8
45 N.	9 45	45 N.	9 45	45 S.	9 30	45 S.	9 30	9
56¼ N.	8 45	56¼ N.	9 0	56¼ S.	8 15	56¼ S.	8 15	8
67½ N.	7 0	67½ N.	7 0	67½ S.	6 30	67½ S.	6 15	6
78¾ N.	4 15	78¾ N.	4 15	78¾ S.	3 45	78¾ S.	3 45	3
North.	0 0	North.	0 0	South.	0 00	South.	0 00	0

Distance of Compass from the Muzzle, 3 feet; Depression, 3° 38′.

East.	0° 0′ E.	West.	0° 0′ W.	East.	0° 0′ W.	West.	0° 0′ E.	0°
11¼ N.	2 30	11¼° N.	2 45	11¼° S.	2 30	11¼° S.	2 30	2
22½ N.	5 0	22½ N.	4 45	22½ S.	4 45	22½ S.	4 30	4
33¾ N.	5 45	33¾ N.	6 0	33¾ S.	5 45	33¾ S.	5 30	6
45 N.	6 30	45 N.	6 45	45 S.	6 30	45 S.	6 30	6
56¼ N.	6 15	56¼ N.	5 45	56¼ S.	6 0	56¼ S.	6 0	6
67½ N.	4 15	67½ N.	4 30	67½ S.	4 30	67½ S.	4 30	4
78¾ N.	2 30	78¾ N.	2 30	78¾ S.	2 30	78¾ S.	2 30	2
North.	0 0	North.	0 0	South.	0 00	South.	0 00	0

* The weight of the gun itself, 51 cwt. 1 qr. 9lb.; estimated weight of the wheels, trucks, appendages, 6 cwt. 2 qrs. 19lb.; making the entire mass 58 cwt.

73. The computed deviations in the last columns of the two preceding tables were obtained by first finding from the observed deviations the mean ratio or value of $A = \frac{\sin 2\lambda \cos l}{\tan \Delta}$, and then using it as a constant co-efficient, (A) in the expression $\tan \Delta = \frac{\sin 2\lambda \cos l}{A}$. The same might likewise be done, by saying,

"As the rectangle of $\sin 2\lambda \cos l$ (corresponding to any position of the compass),

To $\sin 2\lambda' \cos l'$ (answering to any other position),

So is the tangent of the deviation in the first instance,

To that in the second."

For example; the latitude and longitude corresponding to 45° or N E, is lat. 13° 30′; long. 43° 18′; and the same answering to one point from the east, is lat. 3° 44′, and long. 10° 48′: therefore,

As sin 27°. cos 43° 18′ : sin 7° 28′. cos 10° 48′: tan 6° 30′ : tan 2° 30′, which latter is exactly the deviation found by observations in Table II. The former of the above methods, however, is to be preferred, because it is made to depend upon a mean result instead of adopting any one in particular, as must be the case in the latter.

As the reader may not find so close an agreement between the computed and observed results in the

preceding table as he would probably have anticipated, it may be proper to observe, that some allowance must be made in this respect, on account of the nature of the division of the lower circle, by which the direction of the trail was ascertained: but, on the other hand, as the diameter of that circle was considerable, viz. 10 feet 6 inches, (Art. 69,) a trifling discrepancy in the length of the divisions is the less perceptible, although it is impossible to say that an angular error of a few minutes may not exist.

74. In all ships the compass is placed considerably above the common centre of attraction of the guns, and in most, perhaps, so much above it, that the line joining that centre and compass never intersects the *plane of no attraction*, in this or any higher latitude, although that intersection must necessarily take place in approaching towards the equator. In order to make some observations of this kind, I caused the gun to be brought truly horizontal, thereby raising the compass above the common centre of attraction; then, placing the former at 30 inches distance from the muzzle, I caused it to be traversed round in the manner above described, and the following are the results of these experiments:

Trail of the Gun.	Deviation.	Trail of the Gun.	Deviation.
S 0 E	0° 0′ West.	S 0 W	0° 0′ East.
S 11¼ E	2 45 Do.	S 11¼ W	2 45 Do.
S 22½ E	5 0 Do.	S 22½ W	4 30 Do.
S 33¾ E	6 15 Do.	S 33¾ W	6 15 Do.
S 45 E	6 45 Do.	S 45 W	6 30 Do.
S 56¼ E	6 15 Do.	S 56¼ W	6 0 Do.
S 67½ E	3 0 Do.	S 67½ W	2 45 Do.
S 78¾ E	0 0 Do.	S 78¾ W	0 30 West.
East.	4 30 East.	West.	5 0 Do.
E 11¼ N	7 45 Do.	W 11¼ N	8 0 Do.
E 22½ N	10 30 Do.	W 22½ N	11 0 Do.
E 33¾ N	11 45 Do.	W 33¾ N	12 0 Do.
E 45 N	11 30 Do.	W 45 N	11 45 Do.
E 56¼ N	9 30 Do.	W 56¼ N	10 15 Do.
E 67½ N	7 0 Do.	W 67½ N	7 15 Do.
E 78¾ N	3 45 Do.	W 78¾ N	3 45 Do.
North.	0 0	North.	0 0

75. The gun being now elevated 7° 45′, the following observations were made while the trail was traversed through one half circle only, viz. from North to South:

Trail.	Deviation.	Trail.	Deviation.
S 0 E	0° 0′	E 0 N	12° 30′ East.
S 11¼ E	1 45 West.	E 11¼ N	15 45 Do.
S 22½ E	1 30 Do.	E 22½ N	16 30 Do.
S 33¾ E	2 15 Do.	E 33¾ N	16 15 Do.
S 45 E	1 0 Do.	E 45 N	14 30 Do.
S 56¼ E	1 45 East.	E 56¼ N	11 45 Do.
S 67½ E	5 0 Do.	E 67½ N	8 30 Do.
S 78¾ E	8 15 Do.	E 78¾ N	5 15 Do.
East.	12 30 Do.	North.	0 0

76. As in my former experiments the gun was depressed 3° 45′, in order to bring the pivot of the compass into the same horizontal plane with the centre of attraction, we may consider, that when the gun was laid horizontally, the compass was elevated about the same quantity above the centre of attraction; and if we compute at what distance from the east or west points, a small circle parallel to the horizon would intersect a great circle inclining at an angle of 19° 30′, we shall find that distance to be very nearly 11¼° on the horizontal circle; and exactly at this point, we find by the first of the preceding tables the deviation became zero; it being 4° *east* at the east point, and 3° 0′ *west*, at ESE, and zero at E by S.

77. In the same manner, the gun being elevated 7° 45′ in the last experiments, and depressed 3° 45′ in the first, we may consider the compass as being elevated about 11½° above the centre of attraction; and therefore the deviation ought to become zero at that point, where a small circle parallel to the horizon, and distance from it 11½°, intersects the oblique circle of no *attraction*. Without entering into any computation on this head, it is obvious, by referring to our table of experiments page 37, that the latitude corresponding to 35° from the east, is 11° 2′, and to 40°, from the same 12° 33′; the point therefore at

which the deviation ought to vanish, must lie between the S E and S E by E; which is precisely what is shown by the last table of the foregoing observations: the deviation being 1° *west* at the former point, and 1° 45′ *east* at the latter.

78. These experiments will be, I trust, quite sufficient to satisfy every one, that the same laws which I first obtained from observation on regular masses of iron, are equally applicable to irregular masses, and that they furnish us with the means of computing the local attraction of a ship's guns upon her compass, under all circumstances, and in all parts of the world; at least if (as there is the strongest reason to believe) the plane of no attraction varies its position in different latitudes, so as to be every where inclined to the horizon, at an angle equal to the complement of the dip.

79. When the above article was written, the conjecture here advanced was attended with the highest degree of probability; but it has been most satisfactorily confirmed, since the first edition of this work was published, by Mr. Lecount, of H. M. S. Conqueror, by a series of observations made in a voyage from the Cape of Good Hope to England. The experiments were made on bars, handspikes, mast-rings, and various other iron bodies; from the whole of which the author concludes that "A plane or circle held east and west

(magnetic) *and at right angles to the direction of the dipping needle*, divides the north from the south magnetic effluvia, each lying on that side to which the dipping needle points; and by referring the position of all iron bodies to this plane, the plane of section shall divide the iron into north and south polarity, provided it be of uniform thickness.

"If it be not of uniform thickness, the section must be drawn not through the centre of its length, but through its centre of (gravity) attraction. This plane will, therefore, be vertical at the magnetic equator, and horizontal when the dip is either 90° N or 90° S, and will be inclined proportionally to the dip between these situations." *Lecount, on the Magnetic properties of Iron Bodies.* Without examining here the particular illustrations and ideas of the author, the fact of the change above suggested, actually taking place in different latitudes is, I conceive, fully verified.

It may be proper also to add, that these experiments were made without any knowledge of what had been published in the first edition of this work.

SECTION XI.

ON THE LOCAL ATTRACTION OF VESSELS.

80. Before I enter upon the laws of action in this case, it will be proper to offer a few remarks, and make some references to such works as have treated on the subject of local attraction. The first notice of such an effect, that I am aware of, occurs in the voyages of Captain Cook, it having been first observed by Mr. Wales, his astronomer; the cause of the deviation of the needle, however, seems not to have been suspected, and the subject at that time attracted little attention.

The next reference to the local attraction of vessels, and in which the cause is clearly pointed out, is found in Walker's Treatise on Magnetism, published in 1794: it is contained in a report from Mr. Downie, master of H. M. S. Glory, where he says, "I am convinced that the quantity and vicinity of iron in most ships, has an effect in attracting the needle; for it is found by experience that the needle will not always point in the same direction, when placed in different parts of the ship, also it is rarely found that two ships steering the same course, by their respective compasses, will go exactly parallel to each other, yet these compasses, when compared on board the same ship, will agree exactly."

A few years after this, the action of the iron of the vessel was more minutely noticed by Captain Flinders, who was the first to trace its connection with the dip of the needle, and through whose perseverance some attention was paid by Government to the subject, and several experiments were made, by order of the Admiralty, on various ships at the Nore, by which the general fact was established. The subject, however, seems to have been again lost sight of, till Mr. Bain published his valuable little Treatise on the Variation of the Compass; in which the fatal consequences attending this source of error are put in so clear a point of view, as to strike the most indifferent reader. And as at this time our Arctic Expeditions were in contemplation, the local attraction of the vessels was one of the objects to which the attention of the officers was particularly directed. The results of the experiments made in these instances are given by Captains Ross and Parry, in the accounts of their respective voyages, as also by Captain Sabine, in the Philosophical Transactions, Part I. 1819; another memoir on this subject is also published in the same volume, by Captain Scoresby. These statements all agree in establishing the existence of this source of error, and in showing the necessity of some method of correcting it particularly in high latitudes. Let us then examine how far the principles established in the foregoing

sections are applicable to this case. Or how nearly the deviations observed on shipboard agree with our formula

$$\tan \Delta = \frac{\sin 2\lambda \cos l}{A}.$$

I shall select, for making this comparison, the deviations observed by Captains Ross and Sabine, on board the Isabella, in Brassa Sound, Shetland, being those perhaps in which the greatest reliance can be placed, while, at the same time, they embrace the entire circumference, whereby we are enabled to take a mean between every two corresponding deviations, viz. between every two points equally distant from the line of no attraction.*

81. As the above formula has reference to two circles, the one inclining to the horizon at an angle of 15° 38½′ (the dip being 74° 21½′) and its vertical, as also to the position of the common centre of attraction of the vessel; it is obvious that we must proceed by a method of approximation to determine those values of λ and l, and the constant coefficient A, which best agree with the given deviations. In order to this, I begun by assuming different values for the inclination of that line which may be supposed to join the pivot of the needle with the common centre of attraction of all the ship's

* I have, since the above was written, made several experiments of the kind, but I retain the above comparison, as it stood in the first edition of this work.

iron; and after a few trials, I found its inclination to be about 65° with the horizon, or 25° with the vertical, and that the value of the constant co-efficient was A = 7.934 *nearly*.*

These being determined, the other part of the operation is precisely equivalent to the reduction of right ascension and declination to longitude and latitude. The constant declination being 65°, the obliquity 15° 38½′, and the right ascension, the angles given by the direction of the line of no attraction in the vessel, with the magnetic east or west points of the horizon.

82. The following table exhibits the results of these operations, with the computed and mean observed deviations.

* I do not profess to have found either the one or the other of these quantities to the greatest degree of accuracy; for my object being merely to show that the laws in question are applicable to the determination of the deviation on shipboard, I was satisfied with my approximation, when I had brought the errors between the observed and computed results within reasonable limits.

Comparison of the computed, with the observed Deviations, on board the Isabella, off Shetland. Dip, 74° 21½′.

Angle between the line of no attraction, and the east and west points.	Latitude of the centre of attraction.	Longitude of the centre of attraction.	Computed deviation formula, $\tan \Delta = \frac{\sin 2\lambda \cos l}{7{\cdot}934}$	Deviations observed by Captain Sabine. Eas.	Deviations observed by Captain Sabine. West.	Mean of the observed deviations.	Mean Errors.
89° 41′	80° 40′	89° 11′	0° 2′	0° 19′	0° 19′	0° 19′	0° 17′
77 53	79 47	59 54	1 16	1 19	0 26	0 52	0 24
65 43	78 13	35 59	2 5	2 9	1 26	1 47	0 18
53 30	74 40	18 9	3 29	3 4	2 26	2 45	0 43
41 30	71 29	4 29	4 19	3 34	3 26	3 30	0 49
29 30	68 15	6 52	5 0	4 4	4 26	4 15	0 45
17 38	65 8	16 44	5 15	4 34	5 11	4 52	0 23
5 35	62 30	24 35	5 22	5 34	5 46	5 40	0 18
5 40	59 28	33 58	5 14	5 34	5 46	5 40	0 26
16 52	57 5	41 51	4 54	5 34	5 41	5 37	0 43
27 35	55 3	49 9	4 26	4 59	5 11	5 5	0 41
38 2	53 20	56 6	3 51	4 24	4 11	4 17	0 26
48 45	51 53	63 10	3 10	3 34	3 56	3 45	0 35
59 15	50 46	70 2	2 25	3 4	2 56	3 0	0 35
69 7	49 50	76 29	1 40	2 4	1 11	1 37	0 3
79 45	49 29	83 21	0 50	1 34	0 26	1 0	0 10

83. The numbers in the first column of the preceding table are corrected for the mean observed deviation, so as to exhibit the true magnetic bearing of the line of no attraction, assuming (what Captain Sabine has not stated,) that the deviation was eastward while the ship's head was between the east and north, and the east and south, and western in the other semi-circle. The indication of *plus* and *minus* is not sufficient to determine this, as

these signs will change accordingly as the bearing of the object is to the west or east of the north; and the name of the variation will also change, according as the principal centre of attraction is fore or aft of the compass. In most cases that centre is doubtless forward; but in the Isabella, the compass being placed between the main and mizen mast, it is somewhat doubtful on which side that centre might fall: the circumstance also or its being elevated so much above the deck, and its peculiar situation in the vessel, will account for the great inclination that I have found, (65°) for the line joining that centre with the pivot of the needle.

The corrections I have made are on a supposition that the principal focus of attraction was forward: in the other case a different correction will be required; but the results will not be much affected, on account of the deviations being so very nearly equal at equal angular distances from the two extremities of the line of no attraction.

84. I have already observed in the preceding page, that the utmost accuracy has not been aimed at in the above approximations; nor could it probably have been attained, had such been my intention: it is indeed obvious, that as the corresponding observed deviations do not always agree with each other, it is in vain to expect to reconcile them completely with any law that gives the same

deviation to each two positions, equally distant from the same extremity of the line of no attraction. What may be the cause of the discordance in the equi-distant observed deviations, it is difficult to say; but that, in general, there is a tendency to equality, is obvious from the observations of Captain Flinders, Mr. Bain, and those above reported: the aberrations, therefore, where they occur, must be considered as accidental, and due to some cause independent of the general law of the disturbing force.

One cause may be the unavoidable inclination of the vessel during the observations, produced by the wind and tide in opposition to the cable and warp, which necessarily changes in a slight degree the latitude and longitude of the centre of attraction, as referred to the ideal sphere circumscribing the compass, and which will, according to the preceding laws, make a corresponding change in the deviation. A second source of error will arise in the parallax caused by warping the vessel round; which, however, is not likely to be very great. But, on the other hand, it is to be remarked, that the errors which embarrass the results are also, as to their actual magnitude, very inconsiderable; and it is not improbable that the two causes alluded to above may be amply sufficient to account for all the observed irregularities. A third, however, may have some influence, viz. the difficulty of bringing

the ship's head exactly upon the point of the compass at which the observation is intended to be made.*

It appears from what has now been stated, that whether we employ regular iron balls or shells, or an irregular formed body, as a gun with its carriage, &c. or a ship in which the iron is distributed in almost every direction, yet the same laws of action are found to obtain; provided, in the two latter cases, the mass and compass revolve together, so as to preserve the relative position of the latter, and the disturbing body constantly the same.

* Since the above was written, viz. since the publication of the first edition, I have had myself considerable practice in experiments of this kind: first, on board H. M. S. *Leven,* prior to her former voyage; and again very lately. I have likewise performed the same experiments on board H. M. S. Conway, in Portsmouth Harbour, and on board H. M. S. Brig, Barracouta, which accompanies the Leven on her present voyage for surveying the Eastern coast of Africa; and although every possible precaution was taken, (which will be explained in a subsequent section) yet we never could produce a complete coincidence in the corresponding observations: but as circumstances were more favourable, so our corresponding results approximated more nearly to equality.

SECTION XII.

METHOD OF DETERMINING THE LOCAL ATTRACTION OF A VESSEL BY EXPERIMENT.

85 ALTHOUGH the rules given above cannot be considered as involving any great intricacy of calculation, yet they stand in need of one important datum, not easily obtained, viz. the dip of the needle, which renders them, generally speaking, of but little use to practical navigators; I propose therefore to explain, in the present section, a method of reducing the above determination to a mere matter of observation.

My first idea relative to this subject was, that since the guns of a vessel will produce exactly the same deviation of the needle as a smaller mass of iron placed in a similar situation, but so much nearer in proportion as its mass is smaller, that it would be possible to place such a body of iron aft of the compass, as should exactly balance the action of the guns forward, and thereby leave the needle to act in the same manner as if no iron were in its vicinity: but this, unfortunately, I found to be impracticable, without shifting the position of the ball for every different position of the gun or vessel, which would of course render it nugatory. I therefore abandoned this idea for the following, which is simple, and may be easily practised.

86. Since, as I have observed above, the action of the guns is precisely the same as that of a ball of iron of given dimensions, placed in a corresponding situation with respect to latitude and longitude, as referred to the ideal sphere surrounding the compass, it is obvious, by placing such a ball in such a situation, the deviation, instead of being destroyed, will be doubled, and that this will continue to be the case under all circumstances, and in every part of the world, while the ball remains in its place, and the centre of attraction of the guns continue to maintain the same relative position with respect to the compass. Instead, therefore, of fixing the ball, let only its proper place be assigned, and the ball itself laid aside: then, at any time when it is desirable to ascertain the effect of the guns on the needle, apply it in its assigned situation, and observe how many degrees, &c. it attracts the needle out of its prior direction; and just so much will the guns have drawn the same from its true magnetic bearing before the experiment. This being ascertained, and the course of the vessel corrected accordingly, the ball is to be removed and laid aside, till some new circumstance renders its application again necessary.

87. It is to be observed that, strictly, it is not the angle of deviation which is doubled by this experiment, but the tangent of the angle; but, as in small angles the tangents have nearly the same

ratios as their arcs; it will be sufficiently correct to consider it as above stated. If greater accuracy should be thought desirable, let x be the angle of deviation produced by the guns, and a, the angle produced beyond the former by the ball; then we shall have

$$(\tan x + a) = \frac{\tan x + \tan a}{1 - \tan x \tan a} = 2 \tan x.$$

$$\text{Whence } \tan x = \frac{1 + \sqrt{(1 - 8 \tan^2 a)}}{4 \tan a}.$$

This formula may, however, as I have before observed, in most cases be dispensed with.

88. In order to leave nothing doubtful in a case of so much practical importance, and as I had Sir William Congreve's authority to have such apparatus constructed as I should conceive requisite, I ordered a frame work to be affixed to the gun, which should project beyond the compass, whereby I could suspend a ten inch shell in any required position with respect to the centre of the needle. This being prepared, and the ball fixed in the required situation, I repeated my first course of experiments with the ball attached, by traversing the piece through the entire circle. The following table shows the results; in which I have distinguished the angles due to the gun and shell, in order that the reader may see how nearly they approach to equality.

EXPERIMENTS

89. *On an Iron Twenty-Four Pounder, with an attached shell* 96*l.*

Position of trail.	Deviation due to the gun.		Deviation produced by the shell.		Whole deviation		Position of trail.	Deviation due to the gun.		Deviation produced by the shell.		Whol deviati
East.	0°	0′	0°	0′	0°	0′	West.	0°	0′	0°	0′	0°
E. by N.	4	15	3	45	8	0	W. by S.	4	0	4	0	8
E. N. E.	7	30	7	0	14	30	W. S. W.	7	30	7	15	14 4
N. E. by E.	9	0	8	30	17	30	S. W. by W.	9	0	8	45	17 4
N. E.	9	45	9	0	18	45	S. W.	9	30	9	0	18 3
N. E. by N.	8	45	8	15	17	0	S. W. by S.	8	15	8	0	16 1
N. N. E.	7	0	6	45	13	45	S. S. W.	6	0	5	45	11 4
N. by E.	4	15	4	0	8	15	S. by W.	3	45	3	45	7 3
North.	0	0	0	0	0	0	South.	0	0	0	0	0
N. by W.	4	15	4	0	8	15	S. by E.	3	45	3	45	7 3
N. N. W.	7	0	6	30	13	30	S. S. E.	6	30	6	30	13
N. W. by N.	9	0	8	45	17	45	S. E. by S.	8	15	8	15	16 3
N. W.	9	45	9	15	19	0	S. E.	9	30	9	0	18 3
N.W. by W.	9	15	9	0	18	15	S. E. by E.	9	15	9	0	18 1
W. N. W.	7	30	7	0	14	30	E. S. E.	6	30	6	30	13
W. by N.	4	15	3	45	8	0	E. by S.	3	45	3	45	7 3

These results, although they exhibit some small aberrations, are sufficient to show that the principle itself is correct; and that with greater precision, and a more accurate method of suspending the shell, greater accuracy might have been attained.

90. The above experiments had been performed, and the preceding paragraphs written, before I had discovered that the power of an attracting body of iron resided in its surface; and I therefore at that time foresaw an impediment to the practice of this

method on shipboard, in consequence of the mass of iron which I thought would be necessary to produce the desired effect: but having since found, that surface is the principal thing to be attended to, this difficulty is avoided, as a mere globular iron shell, or a simple circular plate of the same metal, is amply sufficient for the purpose. I therefore ordered a double circular plate of iron to be made, 15 inches in diameter, which was found to weigh only 4lb. 13oz., and with this I repeated the preceding series of experiments, and made several others, the whole of which gave the most satisfactory results; and by afterwards attaching the same plate to a ship's binnacle, obtained from his Majesty's dock-yard for the purpose, I found that its power was far greater than would be requisite for doubling the effect of the guns of any vessel in the navy, although applied exterior of the binnacle, and nearly 15 inches distant from the pivot of the needle.*

91. My project being thus far advanced, I presented a memorial to the Lords Commissioners of the Admiralty, soliciting permission to make a trial of my method on board any of his Majesty's ships; to which solicitation I received a reply from the first Secretary, stating, that their lordships,

* In the experiments I have since made on board the several ships mentioned in a preceding note, I have only used plates of one foot diameter.

having had my memorial under consideration, and and having also received a report upon it from the Secretary of the Board of Longitude, they were pleased to grant the permission I requested; and it is in consequence of that permission, I have been enabled to make the experiments detailed in the following chapter.

92. After the communication above referred to had been made to me on the part of the Admiralty, Sir William Congreve very obligingly introduced me to Sir George Cockburn, J. W. Croker, Esq. and some other gentlemen connected with that board, who did me the honour of attending a repetition of a few of my experiments on the twenty-four pounder above-mentioned; and I take this opportunity of returning my best thanks to Sir George Cockburn for the attention he paid to the subject, and particularly for the assurance he made me, that every possible facility would at all times be afforded me by the Admiralty, and that whatever ship might be appointed for the trial, the officers should be instructed to give every possible attention to the subject, and to lose no opportunity of proving its accuracy, and reporting the results, which should be duly communicated to me. Mr. Croker also was certainly much interested in the experiments; and, if I had no reason to flatter myself that he entered the Repository highly prepossessed in their favour, I had the pleasure of

receiving from him, before he left it, his unequivocal opinion, that they were justly entitled to a fair trial in other latitudes, and an acknowledgment of their importance, provided they should be found to succeed. I am also much indebted to him for the order which he afterwards forwarded to Woolwich, permitting me to avail myself of any facilities His Majesty's Dock-yard might afford, in the further pursuit of my inquiries.

Description of a model intended to illustrate the preceding method of correction.

93. Some readers will perhaps form a better idea of the method proposed in the preceding part of this section, by the representation and description of a model I had the honour of presenting to the "Society of Arts,* &c." of which figures (4), (5), are an elevation and plan.

T T is a board or table, in which is fixed an upright spindle S s, which passes through the vessel, and about which it may be turned in any direction at pleasure. D is a brass plate fixed on the deck of the vessel, and divided according to the points

* For this communication the Society in the most unanimous and handsome manner, elected me a perpetual member, and presented me with their gold medal, and a complete set of their Transactions, in 38 volumes.

of the compass, the north and south points being fore and aft.

H is a hand, or index, movable on the spindle; C is the compass, P the correcting plate, and B the rod by which it is attached to the pedestal of the compass. The dotted line passing obliquely downwards from C, is that in which the centre of attraction of all the guns, and of the other articles of iron contained in the model falls, and in this line the centre of attraction of the plate P is also situated, and at such a distance from C, that its power on the needle is equal to that of all other iron at a greater distance. Now, to illustrate the nature of the correction by the model, turn it about on its pivot, till the compass shows north, that is, till the lubber line in the brass compass box, and the north of the card coincide; the vessel is then in the meridian, and the movable index on deck must be set also to the north point. Turn now the vessel on its spindle, till the hand is directed to any other point (as for example East): then if there were no attraction from the iron on board, the compass would read East also; but it will be found to point about E $\frac{1}{2}$ N, which shows the attraction at that point to be about $5\frac{1}{2}^{\circ}$; and in the same way the attraction at any point may be observed, the plate during such time being removed; and if at any of those points the plate be

applied, it will be found to double the quantity of the error.

To illustrate its application in real practice; turn the vessel about (having first adjusted it), till the apparent course by compass, is East, or any other proposed point; and now, to find the true course, apply the plate, and observe how many degrees, &c. it attracts the needle; which, in the model, at East, will be found about half a point, so that the apparent course, by compass, will be now E $\frac{1}{2}$ N; the attraction of the plate having drawn the north end forward about $5\frac{1}{2}°$ or half a point: the iron of the vessel had therefore done the same before the plate was applied; consequently, the true course was E $\frac{1}{2}$ S, and by looking to the index on deck, it will be found that this is actually the course shown. The same will be the case at any other point, except that the quantity of attraction will be different, being most towards the east and west, and less as we approach the meridian. In other parts of the world, however, the east and west will be the points of least attraction, and the greatest will be at the north-east, north-west, south-east and south-west; but still the plate will always continue to give the same attraction as the vessel, and will, therefore, in all places furnish a ready method of correction.

The accurate action of a model is seldom to be expected, and less perhaps in magnetical experi-

ments than in any other. I was therefore very agreeably surprised to find how very correctly this model answered all the conditions which I had found to obtain in the largest vessel.

According to the laws deduced in the preceding sections, it appears that the tangents of deviation are directly proportional to the cube of the linear dimensions of similar iron bodies, and that those tangents are also inversely proportional to the cubes of the distance, all other things being the same: consequently if (as in the model) the tanks, guns, &c. are made proportional to the dimensions of the vessel represented, the linear measures of these will be proportional to the distance of the compass; and therefore the deviations ought to be the same in quantity, as in the vessel at large; and certainly the agreement in this respect is much more perfect than could possibly have been anticipated.

This model is deposited in the Society's rooms, and may be examined by any person introduced by a member.

I have also since constructed a model of a 74 gun ship, which likewise exhibit the experiments in the most satisfactory manner.

SECTION XIII.

ON THE METHOD OF ASCERTAINING THE LOCAL ATTRACTION OF VESSELS.

An Account of some Experiments made on board His Majesty's Ships Leven, Conway, and Barracouta.

94. The first opportunity which presented itself of putting the orders of the Lords Commissioners of the Admiralty into effect, was on board the Leven, which was fitting at Woolwich, for a survey of some part of the western coast of Africa.

The first object, in course, in all cases is to determine the quantity of the local attraction of the vessel; and the method we adopted for this determination in the present instance, will be easily understood by the following extract from my report to the Admiralty:

'The Leven having dropped down to Northfleet on the 15th of April, 1820, I went down on the 17th, for the purpose of making a series of experiments before the guns should be brought on board, these observations were conducted as below.

First, 'Finding that there would be great difficulty in warping the vessel round in the tide way of this place, I proposed, and it was agreed to proceed in the following manner:

'I took on shore an excellent azimuth compass, by Messrs. W. and T. Gilbert, which I had procured for the purpose, as also a theodolite, by Schmalcalder. With the azimuth, the bearing of a distant object was taken, and found to be N. 35° 50′ E., and the theodolite was then adjusted to the same reading, viz. 35° 50′ from zero; by means of which the zero of the theodolite was brought to the true magnetic north, and consequently the bearing of an object might now be determined without any further reference to the needle. It will of course be understood, that the theodolite was fixed immediately over the spot where the azimuth compass was first erected. The latter instrument was now taken on board, for the purpose of the experiments, while Lieutenant Mudge remained on shore to take the bearings of the pedestal,* or pillar, on board with the theodolite.

'The ship now beginning to swing to the tide, the word was given "look out," at which signal Lieutenant Vidal, at the azimuth compass on board, kept Lieutenant Mudge on shore, in the line of the sights, while the latter gentleman kept in the same way, Lieutenant Vidal in the field of the telescope. Being thus prepared, the word

* Captain Bartholomew had ordered a pedestal to be erected just before the mizen mast, as a fixed situation for taking his azimuths during the voyage.

"stop" was given, at which each registered the bearing of the other at the same instant. These bearings, independent of the local attraction of the vessel, ought to have been diametrically opposite, and consequently the difference between the two readings, was the error due to the attraction of the iron on board.

'The first observation being registered, the word "look out," was again given, and then the word "stop," and the same was repeated as often as possible while the vessel was swinging; Lieutenant Baldey taking every time the bearing of the ship's head, by the ship's azimuth compass at the capstan.

'The advantages of this method are, that both bearings, viz., on board and on shore, are made to depend on the same compass, and thus the errors arising from the use of different needles are avoided, as are also those arising from the parallax of a distant object while the vessel is swinging; a source of error which must have attended all former observations of this kind.*

'The only thing actually necessary in this case, is a fine free azimuth compass, those commonly served out to the navy are so sluggish, that it is impossible (while there is no motion in the vessel),

* A reference to fig. 6 may render this description a little more intelligible, by supposing V the vessel in the river R R, and T the station of the theodolite on shore.

to depend upon their settling within 2° or 3° of the true magnetic north.*

'The experiments above referred to, were made before the guns were got on board, but the same were again repeated on the 19th of April, after they had been all shipped. The following are the results of both series of observations:—

EXPERIMENTS

95. *On board H. M. S. Leven, at North-fleet, April* 17 *and* 19, 1820. *By* Mr. Barlow *and the Officers of the above vessel. Dip*, 70° 30′.

Guns not on board.			Guns on board.			
No. of Experiments.	Bearing of Ship's Head.	Difference in bearing or local attraction.	No. of Experiments.	Bearing of Ship's Head.	Difference of bearing or local attraction.	Local attraction shown by the plate.
1	N. 77° 0′ W.	+ 2° 22′	1	N. 71° 0 W.	+ 2° 51′	
2	N. 68 30 W.	+ 2 25	2	N. 64 0 W.	+ 2 07	2 20
3	N. 57 0 W.	+ 1 37	3	N. 57 0 W.	+ 1 39	
4	N. 47 0 W.	+ 1 54	4	N. 47 0 W.	+ 1 45	
5	N. 32 0 W.	+ 1 12	5	N. 31 0 W.	+ 1 39	1 30
6	N. 20 0 W.	+ 1 20	6	N. 24 0 W.	+ 1 10	1 0
7	N. 14 30 W.	+ 0 12	7	N. 15 0 W.	+ 1 19	
8	North.	— 0 15	8	N. 6 0 W.	+ 0 17	0 40
9	N. 5 0 E.	— 0 54	9	N. 4 0 W.	— 0 8	
10	N. 16 0 E.	— 1 32	10	North.	— 0 24	0 0
11	N. 32 0 E.	— 1 43	11	N. 5 0 E.	— 0 11	
12	N. 45 0 E.	— 2 25	12	N. 13 0 E.	— 0 29	0 40
13	N. 52 0 E.	— 2 26	13	N. 23 0 E.	— 0 46	1 0
14	N. 67 0 E.	— 3 15	14	N. 57 0 E.	— 1 27	1 30
15	N. 74 0 E.	— 3 6	15	N. 59 0 E.	— 2 32	
16	N. 83 0 E.	— 2 13	16	N. 72 0 E.	— 2 23	2 10
17	East.		17	N. 80 0 E.	— 2 51	
18	S. 81 15 E.	— 2 34	18	S. 86 0 E.	— 2 11	2 30
19	S. 74 30 E.	— 2 30	19	S. 85 0 E.	— 2 34	2 30

Note 1. The rapidity and force of the tide at Northfleet, at the time of making the experiments on the Leven, would not admit of warping the vessel about point by point, which is doubtless the best way. This, however, is easily done in Portsmouth Harbour, and was the method adopted by Captain Hall, in our experiments on the Conway: reported in the next page.

All the numbers in the following Table marked thus * are those in which two or more observations were made at the same point, and the mean of the two taken. In the others we had not an opportunity of making more than one observation.

Where the apparent, or observed westerly bearing exceeds the true westerly bearing, the error or local attraction is marked + (plus); and where the former is *less* than the latter, the error is marked — (minus). With the ship's head at west, the object on shore could not be seen.

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* Since this sheet has been set up, I am happy to find that the Navy Board has determined upon an improvement in the compasses for His Majesty's ships. I have received instructions for examining all those in store, and am required to make a report of such that are found defective. It is to be hoped therefore that these instruments will soon be placed upon a footing with the other excellent appointments of the British Navy.

EXPERIMENTS.

96. *On the local attraction of H. M. S. Conway, Portsmouth Harbour, July 24, 1820. By* Captain Basil Hall *and* Mr. Barlow, *Royal Military Academy.*

Dip, 70° 30′ 3.									
No. of mean observations.	Direction of Ship's Head.	Observed bearing of the station on shore from the ship, by Captain Hall on board.	Bearing of compass on board, by Mr. Foster on shore.	Local attraction.	No. of mean observations.	Direction of Ship's Head.	Observed bearing of the station on shore by Captain Hall.	Bearing of compass on board by Mr. Foster on shore.	Local attraction.
1	S. b. S.	N. 97° 0′ E.	S 95° 40 W.	—1°20′	17	S. S. E.	N. 97° 0′ E.	S. 97° 15′ W.	+0°15′
2	South.	96 0	94 3	—1 57	18	S. E. b. S.	95 50	96 22	+0 32
3*	S. b. W.	95 20	92 57	—2 23	19	S. E.	94 10	95 16	+1 6
4*	S. S. W.	95 10	92 19	—2 51	20	S. E. b. E.	93 20	94 24	+1 4
5*	S. W. b. S.	94 8	91 0	—3 8	21	E. S. E.	91 0	92 30	+1 30
6*	S. W.	94 2	90 47	—3 15	22	E. b. S.	89 30	91 52	+2 22
7*	S. W. b. W.	93 35	90 15	—3 20	23	East.	87 50	91 15	+2 25
8	W. S. W.	93 30	88 32	—4 58	24	E. b. N.	85 0	89 5	+4 5
9*	W. b. S.	92 10	87 32	—4 38	25	E. N. E.	83 20	86 34	+3 14
10	West.				26	N. E. b. E.	82 10	85 31	+3 21
11	W. b. N.	88 0	84 25	—3 35	27	N. E.	82 15	84 58	+2 43
12	W. N. W.	86 35	83 12	—3 23	28	N. E. b. N.	83 0	85 13	+2 13
13	N. W. b. W	85 20	82 27	—2 53	29	N. N. E.	85 50	88 4	+2 14
14	N. W.	83 25	81 46	—1 39	30	N. b. E.	84 40	85 47	+1 7
15*	N. W. b. N	84 17	82 7	—2 10	31*	North.	83 0	83 7	+0 7
16*	N. N. W.	83 35	82 3	—1 32	32*	N. b. W.	82 28	81 38	—0 50

Experiments on board His Majesty's Ship Leven, (Captain OWEN), *and on her consort, His Majesty's Brig Barracouta,* (Captain CUTFIELD), *previous to the departure of these vessels for the Eastern Coast of Africa.*

97. The Leven on her return from the western coast of Africa, having been ordered to survey the eastern coast of that continent, and the command of her given to Captain Owen, who deservedly enjoys the reputation of one of the most able scientific officers in the navy; I was very happy in having this new opportunity of submitting my method to the test of experiment, particularly, as the vessel was proceeding to a part of the globe, where the dip of the needle is south; and where consequently the trial will be the most conclusive.

Captain Owen had done me the honour of attending a few of my experiments, some time before his appointment, and had taken much interest in their success, he was now therefore anxious to have them made on board his vessel, under his own personal inspection. I accordingly went down to Northfleet, on the 13th January of the present year (1822), for the purpose of superintending the observations, which were made at the same time on board the Leven, and on her consort the Barracouta.

We proceeded nearly as in the cases above,

except that our land station being at a greater distance we used signals by flags, instead of calling to each other; warps were also employed in this instance, and we were thus enabled to steady the vessel on any proposed point, and to take the observation on board with more precision.

A second situation was also taken for a compass, a little forward of the foremast, where a pillar was fixed, and the two compasses placed under like circumstances, except their different situations in the vessel. Another compass was also placed on the gun-room table, with which the ship's head was registered every time it was taken with the other two.

The following are the results of these experiments :

EXPERIMENTS

Made to ascertain the quantity of Local Attraction in H. M. S. Leven, on Two ompasses, one forward and one aft, under the direction of Captain Owen, *nuary* 15, 1822, *at Northfleet.*

After Compass.				Fore-mast Compass.				Ship's head by compass on gun-room table.
head by after compass.	Bearing of Lieut. Owen at shore station from after compass.	Bearing of Lieut. Vidal at after compass from shore station.	Difference or local attration.	Bearing of Ship's head by fore-mast compass.	Bearing of Mr. Daniel's at shore station from fore-mast compass.	Bearing of Lieut. Boteler at fore-mast compass from shore station.	Difference or local attraction.	
′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
00 E.	S. 34 30 W.	N. 34 23 E.	+ 0 7	N. 5 00 E.	S. 33 30 W.	N. 32 20 E.	— 1 10	— —
00	30 30	34 6	—3 36	18 30	Intercepted by the fore-mast.	33 30	— —	N. 17 30 E.
00	30 30	33 30	—3 0	21 30		32 40	— —	— —
00	29 30	34 3	—4 33	28 30		33 30	— —	25 30
00	30 00	34 23	—4 23	29 30		34 00	— —	26 30
00	33 30	35 52	—2 32	36 30		34 12	— —	33 00
30	29 30	34 43	—5 13	51 00		34 47	— —	— —
00	29 00	34 40	—5 40	— —		35 10	— —	— —
00	28 50	34 35	—5 45	— —		35 17	— —	— —
00	29 00	34 19	—5 19	78 0	43 15	35 54	+ 7 21	72 00
00	26 00	33 47	—7 47	84 30	43 15	35 45	+ 7 30	N. 89 00 E.
00	26 30	33 25	—6 55	83 00	43 0	35 26	+ 7 34	S. 88 00 E.
st.	27 20	34 11	—6 51	— —	— —	— —	— —	— —
00 E.	27 00	33 01	—6 01	S. 67 30 E.	42 30	34 47	+ 6 43	73 00
00	27 40	32 45	—5 45	56 15	41 0	34 20	+ 6 40	65 00
00	27 10	32 28	—5 18	50 30	40 30	34 27	+ 6 13	58 30
00	27 0	32 25	—5 25	47 15	39 30	34 12	+ 4 48	52 30
00	26 20	31 57	—5 37	39 30	38 45	33 45	+ 5 00	45 30
00	28 00	31 56	—3 56	38 30	37 30	33 40	+ 3 50	41 30
00	28 00	31 45	—3 45	33 15	37 00	33 23	+ 3 37	37 30
00	27 00	30 57	—3 57	20 00	— —	27 00	— —	— —
uth.	32 00	31 14	+ 0 46	S. 5 00 E.	31 30	30 00	+ 1 30	S. 6 00 W.
00 W.	— —	— —	— —	S. 7 00 W.	25 30	31 23	— 5 57	9 00
00	34 20	31 05	+ 3 15	12 30	25 30	30 50	— 5 20	13 00
00	34 00	31 06	+ 2 54	18 00	24 30	30 50	— 6 20	21 00
00	34 30	31 02	+ 3 28	26 00	23 00	30 31	— 7 31	28 30
00	35 00	31 09	+ 3 51	31 30	22 30	30 38	— 8 08	34 00
00	35 40	31 36	+ 4 4	37 30	22 30	30 49	— 8 19	43 30
00	34 00	— —	— —	45 00	21 30	30 58	— 9 28	50 30
00	35 00	31 23	+ 3 47	— —	22 30	29 30	— 7 00	— —
00	38 50	33 24	+ 5 26	57 30	21 30	32 00	— 10 30	65 35
00	40 20	34 05	+ 6 15	68 30	22 00	32 54	— 10 54	75 30
est.	40 00	34 36	+ 5 24	77 30	22 30	32 36	— 10 06	S. 87 30 W.
4 00W.	41 10	35 00	+ 6 10	N. 87 00W.	21 30	32 56	— 11 26	N. 76 00W.
00	39 50	35 22	+ 4 28	77 00	22 00	33 02	— 11 02	64 30
00	39 40	35 29	+ 4 11	58 30	24 15	33 15	— 9 00	50 00
00	40 30	35 32	+ 4 58	50 00	25 30	33 08	— 7 38	48 00
00	— —	— —	— —	— —	— —	— —	— —	— —
00	39 10	— —	— —	27 30	30 45	36 00	— 5 15	— —
00	38 30	37 30	+ 1 0	19 00	30 45	35 22	— 4 37	— —
00	38 00	37 57	+ 0 3	15 00	31 30	35 43	— 4 13	10 30

(Signed) A. H. Vidal,
First Lieutenant of H. M. S. Leven.

EXPERIMENTS

99. *Made to ascertain the Local Attraction of H. M. S. Barracouta, at Northfleet, January* 15, 1822.

Direction of ship's head.	Bearing of shore station from compass on board, Lieutenant Mudge.	Bearing of compass on board, from station on shore, Mr. Durnford.	Local attraction.	Direction of ship's head.	Bearing of shore station from compass on board, Lieutenant Mudge.	Bearing of compass on board from shore station, Mr. Durnford.	Local attraction.
	° ′	° ′	° ′		° ′	° ′	
N. b. E.	S 37 00 W	N 37 00 E	0 0	North.	S — — W	N — — W	
N. N. E.	34 40	38 40	— 4 0	N. b. W.	— —		
N. E. b. N.	35 40	39 10	— 3 30	N. N. W.	48 10	41 30	+ 6 40
N. E.	34 40	41 20	— 6 40	N. W. b. N.	51 00	42 00	+ 9 00
N. E. b. E.	33 00	42 20	— 9 20	N. W.	— —	44 47	— —
E. N. E.	— —	— —	— —	N. W. b. W	52 30	44 20	+ 8 10
E. b. N.	— —	— —	— —	W. N. W.	56 40	43 40	+ 13 00
East.	26 20	42 40	— 16 20	W. b. N.	55 40	43 30	+ 12 10
E. b. S.	27 20	40 40	— 13 20	West.	54 00	40 59	+ 13 01
E. S. E.	25 00	39 30	— 14 30	W. b. S.	54 40	43 00	+ 11 40
S. E. b. E.	32 00	38 00	— 6 00	W. S. W.	49 18	41 30	+ 7 48
S. E.	33 00	36 00	— 3 00	S. W. b. W	— —	— —	— —
S. E. b. S.	32 00	35 30	— 3 00	S. W.	— —	— —	— —
S. S. E.	32 50	35 00	— 2 40	S. W. b. S.	— —	— —	— —
S. b. E.	— —	— —	— —	S. S. W.	— —	— —	— —
South.	36 00	34 58	+ 1 2	S. b. W.	— —	— —	— —

(Signed) WM. MUDGE,

The points omitted, with the exception of the N. W., were those on which the vessel could not be brought, in consequence of the wind and tide; the observation (on board) at the N. W. is omitted, there being obviously an error in registering the result.

Remark.—The great difference in the amount

of the local attraction of the Leven, this time and the former is doubtless attributable to her being fitted with a new patent capstan, the spindle of which is about eleven feet long, and its mean diameter not less than five inches; it commences from the capstan head, passes through the upper deck and gun deck, and steps into a carling below the beam. Being vertical, the power on the compass is very considerable, although the station for observation is taken as far aft as possible.

In the Barracouta, the spindle is of the same length, and the place for observation is necessarily, from the small dimensions of the vessel, proportionately nearer to it.

Description of the Correcting Plate, Method of adjusting it, &c.

100. The plate which I have employed in all the preceding experiments, as well as those which I have sent with Captain Parry, in the Fury, and with Captain Sabine, who is proceeding for the purpose of experiments to the Island of Ascension, in the Iphigene Frigate, have been double; viz., each consists of two thin plates of iron screwed together, in such a manner as to combine any strong irregular power of one plate, with a corresponding weak part of another, by which means a more uniform attraction is obtained; I am not, however,

certain that these precautions are necessary, if iron weighing about 6lb. to the square foot is made use of; but with thinner plate iron, viz., of about 3lb. to the foot it is requisite, not only for the purpose above stated, but to prevent any accidental bending by a fall or otherwise.

The plates are 12 or 13 inches in diameter, having a hole in their centre, through which is passed a brass socket, with an exterior screw; a brass nut about an inch and a half in diameter, screws on the exterior of each end of the socket, compressing thereby the plates together, with an interposed thin circular piece of board, which is intended to increase somewhat the thickness of the plate, without increasing its weight. It appears also that the compound plate is more powerful when the two, of which it is formed, are thus separated from each other.

In order to render the union the more permanent, I have also fixed the plates to each other by two or three small iron screws near their edges.

101. The plate being thus prepared, the next object is to ascertain its proper situation in the ship; for which purpose we may proceed as follows. Let a box or log of wood, having no iron about it, as AB (Fig. 8.) be taken on shore, and let holes be bored in it, at 8, 9, 10, 11, &c., inches from its upper part, to receive the brass or copper horizontal rod R, which is to carry the plate. This pin

being inserted in one of the holes, and the compass placed securely on the top of the box or log, turn the latter with the pin successively to the several points of the horizon, and by attaching and removing the plate, observe its power of attraction. If the results thus obtained agree with those observed on ship board, we have the right position of the plate; but if this should not be the case (as is most likely to happen), shift the height and distance of the plate, and repeat the experiments again, and after a few trials we shall be able to obtain the same attraction with the plate, as was observed in the vessel.

102. This being done, measure carefully the distance of the plate from the vertical, passing through the pivot of the needle, and its depth below the plane of the card, and cause a hole to be bored, and a socket to be introduced into one of the legs of the tripod, used for the azimuth compass on board, so that when the brass pin is inserted as shown in Fig. 7., the centre of the plate may be at the same depth below, and distance from the vertical passing through the pivot of the compass as was determined on shore, and it will be the fixed situation required. If a fixed pedestal be used on board for the azimuth compass, which is to be preferred, the same direction of course apply to this as to the tripod. The plate and pin are both moveable, and are laid aside except at the time of observation.

It may be proper to observe, that as from the unavoidable errors in observations, it is probably impossible to get the plate to give the same attraction as the vessel at every point. The object ought to be to take a mean between the deviations at (S. E., S. W.), (N. E., N. W.), (E. W.), (N. S.), and if the mean at those points in the ship, and with the plate on shore agree, the other errors will be inconsiderable.

103. After all, however, this is by far the most difficult part of the experiment, and the best way is undoubtedly to purchase a plate already corrected, that is, a plate whose attraction has been found experimentally for several distances and positions; the results of which are given in a table with it, so that having obtained the local attraction of the vessel; the same attraction may be selected out of the table above-mentioned; and opposite to it will be found the proper corresponding height and distance.*

It is to be observed, however, that these *tables* only apply to experiments made in these latitudes; viz., in any British harbour, but they will not be applicable if the experiments are made in a port where the dip of the needle is very different from what it is in London. That is, the experiments on the plate, and on the vessel ought always to be made in the same place.

* Plates, with the requisite tables are sold by Messrs. W. and T. Gilbert, 148, Leadenhall-street.

Method of using the Plate on ship board.

104. In the former edition of this work, I proposed to apply the plate to the binnacle compasses, but on the suggestion of the officers of the Leven, I changed the application of it to the azimuth compass, as being more susceptible of nice observation, besides possessing advantages not to be obtained in the other case.

The following are the directions I left with the officers of the above ship for using the plate:

105. "When an azimuth, or amplitude of the sun, or other heavenly body, is taken for the purpose of determining the variation; the observations are to be first made in the usual way, and then immediately repeated with the plate attached, and the difference between the two bearings will be the local attraction.

"For example:—Suppose the mean of the first series of observations to give the bearing 67°, and the second with the plate attached 70° 30′;

Then,	70° 30′	2d mean	from 67° 0′	
	67 00	1st mean	take 3 30	
	3 30	local attraction.	63 30	correct azimuth.

"Again, let the amplitude by common observation be 13° 30′, but with the plate only 10° 30′, then

13° 30′	1st mean	to	13° 30′	
30 10	2d mean	add	3 0	
3 0	local attraction		16 30	true compass amplitude.

Note.—In all cases, when the first observed bearing of an object is diminished by the plate, the difference or local attraction is to be added to the first bearing; and when the first angle is increased by the plate, the difference is to be subtracted.

106. "It is to be observed, that this supposes the plate to be applied immediately after the mean of the first series of observations has been obtained, and before any considerable change has taken place in the azimuth of the celestial body. To avoid every possible chance of error from this cause, the officers of the above ship, computed their variation for both series of observation;—viz., with, and without the plate, and the difference gave them the local attraction. This is some additional trouble, but the result is proportionately more correct.

The above are all the directions necessary for using the plate, and they are such as cannot, it is presumed, be misunderstood by any seaman entitled to the character of a practical navigator."

An Account of the Experiments made on board the Leven, in her late Voyage to the Coast of Africa.

107. It will be seen by examining the first series of experiments on this vessel, that the local

attraction was very inconsiderable;—viz., but little more than half that of the Conway, a ship of nearly the same class, and scarcely one third of what it is now in the same vessel; consequently the results are not so striking as we might otherwise have expected. At the same time, however, it should be observed, that notwithstanding the local attraction was so small in the situation selected for the azimuth compass, which was used as a constant standard of comparison; yet the binnacle compasses were strongly affected by the action of the vessel, the two frequently differing from each other $7\frac{1}{2}°$ or $8°$. Therefore, although but little correction could be obtained by the plate in determining the variation, yet considerable difference was found between the apparent and correct courses by the binnacle compasses; which is indeed one of the chief objects of the experiments.

To the officers of this vessel I am under the greatest obligation for the pains they took to give the method I had proposed the most impartial trial, and for the results they furnished me with on their return. From Captain Baldey (who had succeeded to the command of the vessel during the voyage, on the death of Captain Bartholomew), I received a copy of the mean results of nearly 100 series of observations, with and without the plate; the local attraction at each, the variation, the latitude, longitude, &c.; and about the same number from Lieutenant

Vidal, Lieutenant Mudge, and from the Master, Mr. Higgs. It would be irksome to the reader to give him these series of observations, it will be sufficient to say, that they are not only highly satisfactory to me, but also to the officers themselves, as will appear from the following letter addressed to me by Captain Baldey on the return of the vessel.

No. 1, *Bath-place, New Road,*
Aug. 15, 1821.

DEAR SIR,

I HAVE left for you in the care of Lieutenant Mudge, a copy of the results of a series of observations, made by me on board His Majesty's ship Leven, with your correcting plate, attached to Gilbert's patent azimuth compass, the original having been already transmitted to my Lords Commissioners of the Admiralty, and I beg to congratulate you on the success which has attended the experiments.

You will perceive that in several instances our binnacle compasses differed from each other a half to three quarters of a point; which, however, we were always enabled to correct by your plate, and in all cases our place by reckoning, when thus corrected, agreed as closely with the observations as we could have reason to expect. Indeed little need be said to shew how very erroneous a place by reckoning must be found, after a run of several

hours 5, 6, or 7 degrees out of the supposed course. At sea such an error, although very considerable, is not perhaps of much importance, but in making land, in entering a channel, and in narrow seas; it might be, and doubtless has been frequently attended with the most fatal consequences.

Under this impression, and being convinced from experience of the simplicity and efficacy of your experiments, I beg that you will make any use of this letter, which you think will be of the greatest service in bringing your method of correction into general practice. I have only further to add, that I have no doubt that the officers, who at present remain on board the Leven, will allow you every facility you may desire, to make such extracts from the log, as you may think essential for pointing out more particularly the advantages of your mode of correction.

I am, Dear Sir,

Your sincere well wisher,

W. BALDEY.

To P. Barlow, Esq., Woolwich.

108. The following is an instance of the kind alluded to above, which I am indebted for to Lieutenant Mudge; it is contained in a letter from that gentleman, dated Santa Cruz, Teneriffe, May 28, 1820.

"On the 22d of May, at noon, we were in latitude 41° 46′ N., and long. by Chronometer 9° 53′ W. Taking this as our departure, we sailed by the starboard compass S. 46° W. 183 miles; this placed the ship on the 23d (allowing the variation 21° W.) in lat. 38° 58′ N., and long. 11° 26′ W. Whereas, the observation at noon for latitude, and sights in the morning for longitude gave 38° 39′ N. and long. 10° 58′ W. So great a difference in 24 hours was attributed to a current, till I compared the starboard or steering compass, with the one with your plate, when I found no less than 7° error, to be subtracted from the course steered, making the true course S. 17° W., instead of S. 24° W., which had been taken as correct. By allowing the 7° which we found subtractive from the course, our latitude was by reckoning 38° 41′ N., and long. 11° 02′ W., which agree with observation as closely as we can ever expect it to do under any circumstances."

109. Such is the present state of this method of correcting the local attraction of vessels; and here I must take my leave of the subject, on which I have already bestowed much time, and have incurred some pecuniary charge in carrying the experiments into execution. I have, I trust, shewn very clearly by the results reported in the preceding part of this section, particularly in those of the

Barracouta, that the errors arising from the local attraction, are of such a nature and amount, as to require correction. I have also explained a simple method by which this is proposed to be effected; and I have given testimonials of its efficacy, in a case where it was submitted to trial during a voyage of sixteen months.

This, perhaps, might be thought sufficient; but I have likewise attempted, (and I trust successfully,) in the following sections, to reduce the laws of induced magnetism to certain and definite principles, and have thence drawn a mathematical demonstration of the accuracy of the proposed method, and of its applicability in all parts of the world.

Fortunately mathematical demonstrations are not subjects admitting of a difference of opinion; therefore, if some error has not slipped into my premises, or into the steps of the investigation, my conclusions must be admitted, and consequently also the truth and generality of the method in question. If, on the other hand, any inaccuracy be detected in the process I have followed, I ask only to have it fairly and openly stated, and with this request I leave the proposition to the attention of those public boards, whose province is to assist and promote improvements in navigation, and who must be well aware that their authority and en-

couragement, as well as their mere concurrence, is necessary to effect any important change in the practice of that art.

SECTION XIV.

ON THE EFFECTS PRODUCED IN THE RATES OF CHRONOMETERS, BY THE PROXIMITY OF MASSES OF IRON.*

110. It having been ascertained during the voyage made by Captain Buchan to the Arctic regions, in the year 1818, that the rates of the chronometers were considerably different on board and on shore; and this change having been attributed to the iron of the vessel,† I felt very desirous, first, of ascertaining whether the proximity of a mass of iron had actually any effect in changing the rate; and, secondly, supposing this to be the case, to determine, if possible, the laws and principles by which that action was governed.

* From a Memoir by the author, published in the Philosophical Transactions for 1821.

† See a Memoir by Mr. Fisher, who accompanied Captain Buchan, in the Philosophical Transactions for 1820, Part II. and a very ingenious paper on the same subject, by the late Mr. Varley, in Vol. I. of the Philosophical Magazine.

I accordingly, through the kindness of some of my friends, procured the loan of six excellent chronometers, besides one or two others, which upon trial were found to have too wide and irregular rates for my purpose. Having procured these, and made the requisite preparations, I began my series of observations on them on the 11th of March, 1821, and continued them daily till the 25th of May; when, having obtained a considerable number of results, they were discontinued. It will however be proper, before I proceed to the detail of particulars, to explain the views I had formed on the subject, and the principles upon which I conducted the experiments.

111. I conceived, that if such an effect as that described by Mr. Fisher, were generally produced on the rates of watches and chronometers, it must arise from the spring, or some part of the balance having become magnetic, and the consequent attraction of the iron upon it. But this would lead us also to conclude, that accordingly as the balance was placed in this, or that direction, with respect to any given mass of iron, the rate of the chronometer would be accelerated or retarded, and not uniformly accelerated, as would seem to be the case by Mr. Fisher's observations. Or rather perhaps I ought to say, that a different direction of the balance would alter the arc of its vibration, from greater to less, or from less to greater: but

it would still depend upon the original adjustment of the machine, whether the result would be to accelerate or to retard its action; that is to say, it would depend upon the contingency, whether the chronometer had a tendency to gain, or lose, in short arcs, which I am informed is nearly an equal chance, if it proceed from the hands of a scientific workman; but that, in general cases, the probability is, that the watch will lose in large arcs, and gain in small ones.

112. The experiments and observations which Mr. Fisher describes as having been made with a strong bar magnet, brought within two inches of the balance, I consider to be perfectly distinct in their nature from those which were made by him on board and on shore at Spitzbergen; for a magnet of such power, brought within the distance of two inches of any small piece of steel, will, whether the latter be previously magnetic or not, impress upon it a strong temporary derangement of its latent magnetism, and give to the part nearest the magnet, a contrary pole to that by which it is opposed; and consequently, there will exist between the balance and the magnet, a strong power of attraction, sufficient to cause that acceleration so strongly indicated in Mr. Fisher's experiments; and this will be the case whichever end of the magnet is opposed to the balance, and to whatever part of the latter the application is made;

because, in this instance, the effect does not depend upon the previous magnetic state of the balance, but upon that temporary state excited by the proximity of the magnetic bar, and which ceases when the bar is removed.

115. This explanation will not, I conceive, apply to the action of plain unmagnetized iron; for notwithstanding, according to the present received doctrine of magnetism, every mass of soft iron becomes a temporary magnet by induction from the earth; yet I am not aware that ever any particular action has been discovered between two pieces of iron, whether hard or soft, which had not previously acquired a polar quality; the largest mass of iron, for instance, will not, that I am aware of, attract and give direction to the lightest and most freely suspended needle of soft iron, or of unmagnetized steel.

Now, if this be admitted, it necessarily follows, that plain unmagnetized iron can only be supposed to act on the balance of a chronometer, when that balance has acquired a polar or directive quality; and then, as I have already stated, it will have a tendency to produce an acceleration, or retardation, according to its position with respect to the balance, and the previous adjustment of the machine.

If this be actually the case, it may probably appear singular, that all Mr. Fisher's chronome-

ters were accelerated; but it is not much less so, that five out of the six which I used in my experiments were as decidedly retarded. It will likewise, after examining my experiments, be difficult to account for that high degree of acceleration noticed by Mr. Fisher; for it will be seen, that, although I approximated some of my chronometers to within two or three inches of the surface of an iron ball thirteen inches in diameter, the utmost effect which I could produce did not exceed 4″ per day; whereas Mr. Fisher makes *his* amount to 8″ or 9″ per day; and yet we can scarcely imagine that he brought his chronometers so closely within the immediate sphere of action of any mass of iron, more powerful than that described in my experiments; indeed we are led strongly to suspect, that the remarkable change in the rates of the nine chronometers of the Dorothea and Trent, reported by Mr. Fisher, must have been produced by some extraordinary cause, not commonly operating on shipboard.

116. I have already observed, that, according to the idea I entertain of the action of iron on the balance of a chronometer, it is actually necessary to conceive, that part of the machine, or at least its spring, to have acquired a certain polar or directive quality, whereby, independent of any other power, the balance would have a tendency to assume a certain direction, when brought within the sphere

of action of a given mass of iron; and the amount of that tendency might, I conceived, be estimated, by counting the number of vibrations which a small magnetized needle would make in a given time, in any assigned situation, near the iron, and comparing the result with the number it would make under like circumstances, and in the same time, when wholly removed from any attracting mass.

117. In order to illustrate this view of the subject a little more particularly, let A B C D (fig. 9,) represent the balance of a chronometer, *s*, *s'* its spring, and let D be that part of the rim which is attracted by the centre *o*, of an iron ball or shell. If now we conceive the spring to be detached from the fixed part of the machine, it will be free, with the balance itself, to take any position. The point D will therefore be attracted towards *o*; and if it be displaced from this position, it will have a tendency to oscillate on each side of the point D; and the number of vibrations which it would make in a given time would serve, if we could obtain such results, to estimate the intensity of action of the attracting body.

But although we cannot detach the balance, for such an experiment, we may still form some idea of the intensity of action, by causing a small magnetized needle to oscillate in the place of the balance, and by counting the number of its vibra-

tions as above described. Indeed there is not much difficulty in estimating, theoretically, the change of intensity due to a certain change in the position and distance of the attracting body ;* but I prefer experiment in this case as more satisfactory to those who may not be able to follow out completely the mathematical investigation on which such a computation must depend. With this previous view of the subject, I began with first ascertaining the time in which forty vibrations were made with a small magnetic needle in different situations with respect to an iron shell eighteen inches in diameter, and at eighteen inches distance from its centre ; the weight of the shell being 496 lbs.

118. But as the degree of intensity, as well as the quantity of deviation, occasioned by the iron ball, has reference, not to the plane of the horizon, but to *the plane of no attraction*, I proceeded with these experiments as follows :

Let S Q N Q′ (fig. 10,) represent the iron shell, or a sphere concentric with it ; Q Q′ its magnetic equator, or plane of no attraction, and *ab*, *cd*, *ef*, &c. parallels of latitude answering to 60°, 45°, 30°, &c. H H′ the horizon, and S N the natural direction of the magnetic action in this place ; the circle S Q N Q′ denoting the plane of the magnetic meridian, agreeably to the division of the magnetic sphere, as described in the preceding part of this Essay.

* See Part. II. Art. 183.

I now first placed the compass at Q, eighteen inches from the centre of the shell, and observed the number of seconds which the needle employed to make forty vibrations; then, still keeping the needle in the circle QQ′, I placed it 30° from Q towards E, or in longitude 60°. I then brought it 30° nearer to E, or into longitude 30°; then to E, or longitude 0°; and so on at every 30° through the whole circle. The same was then repeated in the circles *ab*, *cd*, *ef*, &c.; and by taking the mean of the results for each corresponding situation on each side of the meridian, I obtained the numbers given in the following table:

119. *Table showing the time of making ten vibrations with a fine magnetic needle in different situations, eighteen inches from the centre of an iron shell, eighteen inches in diameter, weighing* 496 *lbs.*

Latitude.	Mean time of making 12 vibrations.						
	Long. 90° N	Long. 60° N	Long. 30° N	Long. 0°	Long. 30° S	Long. 60° S	Long. 90° S
°	″						
90 N	30·25	″	″	″	″	″	″
60 N	26·00	27·25	28·00	29·75	34·50	35·75	38·25
45 N	26·25	27·50	28·25	30·00	35·25	39·25	43·50
30 N	27·25	27·50	29·00	31·25	34·00	41·25	46·50
0	34·50	34·75	34·75	35·50	35·00	35·00	35·00
30 S	46·50	41·25	34·00	31·25	29·00	27·50	27·25
45 S	43·50	39·25	35·25	30·00	28·25	27·50	26·25
60 S	38·25	35·75	34·50	29·75	28·00	27·25	26·00
90 S							30·25

Mean time of making 10 vibrations detached from the iron ball 32″· 50.

These experiments were made with a small steel bar or magnetic needle, finely suspended with untwisted silk in a glass vessel, and some care was taken to get the time as accurately as seemed desirable for the purpose; but as the only intention of the experiments was to have some general ideas of those situations near the ball, where a compass needle would be most affected in its vibrations, and where also, according to my ideas, the chronometer would be most affected in its rate, I did not conceive it necessary to carry these observations to the utmost degree of precision.

120. Every thing being thus prepared, I applied to my friend, the Reverend Mr. Evans, to allow the experiments to be conducted at his observatory, in which was an excellent transit instrument by Troughton, and every thing requisite for conducting them with the greatest accuracy. To this request he very readily assented; and he superintended the observations with the utmost attention, from March 11 to April 30, when, being about to remove to another part of the country, he was obliged to dismantle his observatory, and the experiments, during the rest of the period, were carried on in the same way by myself in the Observatory of the Royal Military Academy.

It does not appear requisite for my present purpose, to give the detail of these experiments at full length, it will be sufficient here to state the

mean results, and to refer those readers who are desirous of more particular information to the volume of Transactions above quoted.

In the following table, the first column describes the situation of the chronometer; the second the mean daily rate; the third the gain or loss, per day in each position; the fourth the number of days the watch was in each situation; and the fifth the proportional magnetic intensity of the iron on the balance, estimated by the number of vibrations which a freely suspended horizontal needle made in the place where the chronometer stood. The intensity of the needle when wholly detached from iron being called 100.

121. *Table of Experiments on the Rates of Chronometers by the proximity of masses of Iron.*

Situation, &c. of the Chronometers.	Daily rate.	Gain or loss per day.	No. of days in each position.	Prop. Mag. intensity
Chronometer denoted by No. I.				
te of Chronometer before it was applied to the ball - - -	— 3″·2	—	14	100
ronometer to the South of the ball 2·1 inches from the floor, and distant from vertical passing through the centre of the ball 17·31 inches, corresponding to lat. 0° long. 90°, distance from centre of ball 18 inches. 12 o'clock, South - -	— 5·6	— 2·4	6	91

Situation, &c. of the Chronometers.	Daily rate.	Gain or loss per day.	No. of days in each position.	Prop. Mag. intensi
Chronometer No. II.				
Rate of Chronometer before it was applied to the ball - - -	+ 6	—	10	10
Ditto ditto - - -	+ 5	—	4	10
Chronometer placed above the ball, height from floor 23 inches, distance from vertical, through the centre 6 inches to the South, corresponding to lat. 90° South, and distance 18 inches. 12 o'clock, North -	+ 6·5	+ 1·5	6	11
Detached from ball - - -	+ 5·8	—	3	10
Ditto ditto - - -	+ 5·2	—	6	10
Chronometer placed to the North of ball, height 10½ inches; distance from vertical 11·31 inches, corresponding to lat. 0° long. 90°, distance 12 inches. 12 o'clock, South.	+ 6·1	+ 0·9	3	9
Detached from ball - - - -	+ 4·7	—	3	10
Placed above the ball; height from floor 17·3 inches, distance from vertical 4 inches to the South of the ball; or lat. 90° S.; distance 12 inches. 12 o'clock, South -	+ 5·0	— 0·2	5	16
Same latitude; distance 18 inches. 12 o'clock, South - - - - - -	+ 4·0	— 1·2	2	11
Detached from the ball - - -	+ 4·3	—	7	10
Chronometer No. III.				
Mean detached rate - - -	+ 0·6	—	14	10
Chronometer placed to the South of the ball, height from the floor 11·3 inches; distance from vertical through centre of ball 6 inches South, corresponding to lat. 90° S.; distance 18 inches. 12 o'clock, South -	— 0·9	— 1·5	8	11
Same situation as above, except the distance being reduced to 12 inches. 12 o'clock, S.	— 0·2	— 0·8	4	16
Placed to the East of the ball; height 6·5 inches; distance from vertical 12 inches; or lat. 0° long. 0°; central distance 12 inches. 12 o'clock, South - -	— 0·9	— 1·5	3	8

Situation, &c. of the Chronometers.	Daily rate.	Gain or loss per day.	No. of days in each position.	Prop. Mag. intensity
Chronometer No. IV.				
ean detached rate - - -	+ 1·5	—	18	100
aced the Chronometer to the South of the ball; height from the floor 11·4 inches; distance from the vertical passing through the centre of the ball 17·3 inches, corresponding to lat. 35° 16′ S, long. 90°, and central distance 18 inches. 12 o'clock, South - - - -	— 0·5	— 2·0	6	126
aced to North of ball; height 10·5 inches; distance from vertical 11·3 inches; or lat. 0°; long. 90°, central distance 12 inches. 12 o'clock, South - - - -	— 0·3	— 1·8	3	94
me situation. 12 o'clock, West -	+ 0·3	— 1·2	3	94
me situation. 12 o'clock, East -	+ 0·8	— 0·7	3	94
me situation. 12 o'clock, North -	+ 0·6	— 0·9	5	94
me situation, but with the 12 o'clock, South, as above - - - -	— 0·6	— 2·1	5	94
aced this Chronometer on pedestal,* South of the plate; distance from vertical through the centre of plate 10 inches; height above centre 10 inches. 12 o'clock, South -	+ 0.2	— 1·3	3	—
This plate was one of those described in the preceding section.				
aced Chronometer to the East of the ball, 2 inches from the floor; distance from vertical through the centre of ball 11·1 inches, corresponding to lat. 20° 45′, long. 7° 45′ central distance 12 inches. 12 o'clock South - - - -	+ 0·2	— 1·3	9	97
aced South of ball 1 inch from the floor, 10 inches from vertical; or lat. 9° 19′ N. long. 90°; central, distance 11·4 inches. 12 o'clock, South - - -	+ 1·1	— 0·4	3	63
tto ditto. 12 o'clock, North -	+ 1·3	— 0·2	3	63

* See art. 125.

Situations, &c. of the Chronometers.	Daily rate.	Gain or loss per day.	No. of days in each position.	Pro Mag intens
Chronometer No. V.				
Mean detached rate - - -	+ 0·9	—	6	10
Placed to the South of ball; height 9·8 inches from floor; distance from vertical 11·3 inches, or lat. 35° 16′ S. long. 90°, central distance 12 inches. 12 o'clock, South -	— 3·4	— 3·6	4	14
Same situation. 12 o'clock, North -	— 3·3	— 3·5	3	14
Same situation. 12 o'clock, West -	— 2·5	— 2·7	3	14
Same situation. 12 o'clock, East -	+ 0·7	— 0·5	3	14
Same situation; but the 12 o'clock turned back to the South, as above - -	— 3·9	— 4·1	5	14
Placed on pedestal* to the South of the plate; height above the centre of the plate 10 inches; and distance from vertical through centre of plate, 10 inches. 12 o'clock, South - - - - - -	— 3·2	— 3·4	4	-
Detached both from ball and plate, to ascertain whether it would return to its former detached rate - - -	+ 0·4	—	8	10
Ditto ditto - - -	— 0·6	—	6	10
Placed to the North of ball; height from floor 6½ inches: distance from vertical 10 inches, corresponding to lat. 19½° N. long. 90°. Central distance 10 inches. 12 o'clock, South - - - -	— 2·1	— 2·3	5	11
Placed North of ball; height from floor 1 inch; distance from vertical 12 inches; or lat. 44° 8′ N. long. 90°; central distance 13·2 inches. 12 o'clock, South -	— 1·4	— 1·6	3	15
Placed to the South of ball; height 8 inches; distance from vertical 10 inches; or lat. 28° 2′ S. long. 90°; central distance 10·1 inches. 12 o'clock, South - -	— 1·7	— 1·9	3	19
Placed South of ball; height 12 inches; distance from vertical, 10 inches; or lat. 48° 30′ S. long. 90°; central distance 11·4 inches. 12 o'clock, South - -	— 2·3	— 2·5	4	20
Same situation. 12 o'clock, East -	— 1·7	— 1·9	4	20
Note. The mean of all the detached rates is	+ 0·2			
* See art. 125.				

Situation, &c. of the Chronometers.	Daily rate.	Gain or loss per day.	No. of days in each position.	Prop. Mag. intensity
Chronometer No. VI.				
These rates were taken before the Chronometer was applied to the ball - -	+ 0·2	—	4	100
Placed to the East of ball; height from floor 2 inches; distance from vertical 11·1 inches, corresponding to lat. 20° 45′ N. long 7° 45′, distance from centre 12 inches. 12 o'clock, South - - - - -	— 1·3	— 0·9	5	97
Similar situation, and at the same distance to the West of the ball. 12 o'clock, South	— 1·5	— 1·1	5	97
Placed to the North of the ball; height 2 inches, distance 14 inches; or lat. 37° 20′ N. long. 90°, central distance 14.6 inches. 12 o'clock, South - - -	— 1·6	— 1·2	3	127
Detached; and the farther observations transferred to the Royal Military Academy -	— 0·6	—	5	100
Ditto ditto - - -	— 0·5	—	7	100
Placed to the South of ball; height 1 inch; distance from vertical 12 inches; or lat. 5° 8′ N. long. 90°, central distance 13·2 inches. 12 o'clock, South - -	— 1·9	— 1·5	3	91
Placed to the South of ball; height 6½ inches; distance from vertical 10 inches; or lat. 19½° S. long. 90°, central distance 10 inches. 12 o'clock, South - -	— 1·4	— 1·0	4	183
Placed to North of ball; height 1 inch; distance 10 inches from vertical; or lat. 48° 18′ N. long. 90°, central distance 11·4 inches. 12 o'clock, South - -	— 1·4	— 1·0	4	169
Placed to North; height 13 inches; distance from vertical 9 inches; or lat. 16° 20′ S. long. 90°, central distance 11 inches. 12 o'clock, South - - -	— 1·2	— 0·8	4	33
Same situation. 12 o'clock, North -	— 1·0	— 0·6	4	33
Note. The mean of all the detached rates is	— 0·39			

Practical deductions from the results of the preceding experiments.

122. The first general conclusion which may be drawn from the foregoing experiments, is, that the rate of a chronometer is undoubtedly altered by its proximity to iron bodies.

Secondly; it appears that it is by no means a general case, that iron necessarily accelerates the rate of a chronometer, as would appear from Mr. Fisher's observations; for five out of the six chronometers which I have made use of, were obviously retarded in every situation in which they were placed. In one instance only, viz. chronometer No. II. there is an indication of acceleration in one situation; but it is more doubtful than the retardation in all the other five.

It is also very obvious from the experiments on Nos. IV. and V., that much depends on the direction of the balance with respect to the iron: thus, No. IV. lost nearly 2″ per day when its 12 o'clock hour mark was turned to the South, and only seven tenths when it was placed to the East; but as soon as the chronometer was returned to its old direction, the loss again became 2″·1 daily. The same occurred in the case of No. V., which lost 3″·6 per day in one direction, and gained 0″·5 in another at right angles to it; and on returning it again to its former direction, the losing rate be-

came $4''\cdot1$ per day, viz. rather stronger than at first. It must be admitted, however, that the same striking difference in the rate, as depending upon direction, was not observed in another instance, when a similar experiment was repeated on the same chronometer. Speaking generally, it also appears, that the greatest effect is produced in those instances where the change in the magnetic intensity is the greatest; but there does not seem to be that uniformity of relation in these cases, that we should naturally have anticipated.

123. As a practical conclusion, it is obvious, that on shipboard, great care ought to be taken to keep the chronometers out of the immediate vicinity of any considerable mass, or surface of iron; on which account, they ought not to be kept in the cabins of the gun-room officers, which are on the sides of the vessel; and probably a strong iron knee, or even a gun, will be found at a very inconsiderable distance from the spot where the watch is most likely, in this case, to be deposited.

In short, it appears from the preceding experiments, that a chronometer ought to be kept as carefully at a distance from any partial mass of iron, as the compass itself.

In support, and in confirmation of the necessity of taking the above precautions, it may not be amiss to state the following fact. A very intelligent seaman, many years a Master in the

Navy, and at present an officer in the Dock-yard at Woolwich, to whom I was describing the nature of my experiments, immediately answered, that they explained a circumstance which he had remarked when he was master of a first rate. He informed me, he always found that his chronometer, which was a very excellent one, had a different rate on board and on shore, amounting to 5″ per day; but as he well remembered that the birth he had selected for it was in his cabin, nearly in contact with an iron knee, he now saw that it was the action of that mass of iron which had caused all his perplexity.

124. Lastly; since it is rendered obvious by the experiments with the plate of iron on Nos. IV. and V. that the power of the iron to disturb the action of the chronometer resides (as in the instance of the compass), on the surface, and as we know, generally, the distance and direction of such a plate, so that its power may be equal to the mean action of the iron of the vessel, we have thence a ready method of ascertaining, before a chronometer is sent on board, whether the effect of the ship's iron will be to accelerate or retard its going; and probably, a very near approximation to the actual quantity of that change may also be predicted.

For this purpose, it is only necessary to have a box or pedestal, as shown in (fig. 8.) pl. iii. in the

side of which a brass pin, is fixed, to carry the iron plate P, and on the top of the box a convenience for placing the chronometer. Then, having taken its rate in the usual way, let it be taken again while the chronometer is placed on the pedestal, keeping the plate, generally, at the distance of about twelve inches from the vertical through the centre of the dial, and its centre about the same depth below the plane of the balance, and the rate thus obtained, will be a very close approximation to the ship rate of the instrument, provided care be taken, when it is removed on board, to keep it out of the immediate action of any partial mass of iron. The plate for this purpose should be a double one, such as I have described in the preceding section.

It should be observed, that the plate is meant as a substitute for the iron forward; and therefore the chronometer when on board, should be placed in the same direction in reference to the ship's head, as it had with respect to the iron plate when its rate was determined; that is, if the 12 o'clock mark of the dial be turned towards the iron plate on shore, then must the same be turned towards the ship's head when taken on board.

125. The following table of the land and sea rates of the four chronometers, whose numbers are specified in it, is taken from the Edinburgh Philosophical Journal, for October 1, 1821. It

is contained in a communication to the Editor, from Lieut. William Mudge, and is copied from the journal of H. M. S. Leven, the rates having been ascertained conjointly by the above gentleman and by Lieut. A. H. Vidal.

Table of the Land and Sea Rates of the Chronometer supplied to H. M. S. LEVEN, *during her voyage to the Cape de Verd Islands, in the year* 1819.

Particulars of Time, place, and year, 1819.	No. 1970, Arnold rate.	No. 498, Arnold rate.	No. 249, Harris & Hutton rate.	No. 503, Arnold rate.
	"	"	"	"
S R. Lisbon, Jan. 2, to Jan. 28.	—	l. 3·70	g. 2·86	g. 7·74
S R. St. Jago, Feb. 8, to Feb. 14.	l. 17·30	l. 1·98	g. 5·53	g. 5·68
S R. Isl. of Sal. Feb. 28, to Mar. 28.	l. 16·27	l. 1·25	g. 6·83	g. 7·29
S R. Ditto, March 28, to April 20.	l. 16·99	l. 0·99	g. 6·82	g. 9·80
S R. Quarl. Island, Ap. 27, to May 4.	l. 17·90	g. 0·26	g. 6·34	g. 10·39
S R. Ditto, May 4, to May 12.	l. 17·66	g. 0·66	g. 6·55	g. 9·68
Mean S R. from the above.	l. 16·95	l. 1·47	g. 6·52	g. 8·47
L R. Maderia, June 20, to July 7.	l. 14·88	g. 1·27	g. 2·00	g. 13·62
L R. Ditto, July 7, to July 17.	l. 13·90	g. 3·85	g. 1·85	g. 14·85
L R. Ditto, July 17, to July 28.	l. 13·72	g. 2·83	g. 3·64	g. 13·82
L R. Ditto, July 28, to Aug. 6.	l. 14·40	g. 2·73	g. 2·84	g. 13·51
L R. Ditto, Aug. 6, to Aug. 24.	l. 13·85	g. 2·60	g. 2·87	g. 13·15
L R. Ditto, Aug. 24, to Sep. 1.	l. 14·23	g. 2·76	g. 2·26	g. 12·90
L R. Ditto, Sep. 1, to Sep. 13.	l. 14·10	g. 3·20	g. 3·50	g. 14·60
L R. Ditto, Sep. 13, to Sep. 19.	l. 14·10	g. 3·30	g. 3·50	g. 14·60
Mean L R from above.	l. 14·17	g. 2·69	g. 2·75	g. 13·80
Difference between mean Land and Sea rates - -	2·78	3·22	3·77	5·33

The Land rates were taken with the Astronomical Quadrant at the house of the British Consul.

SECTION XV.

On the relative magnetic power of different descriptions of iron and steel, at different degrees of temperature.*

126. In the former edition of this work, I limited myself to the experimental deductions of the laws of magnetic action, exhibited by iron bodies, on a magnetized needle, and except a few isolated remarks, I made no attempts at a theoretical investigation. But soon after my Essay was published, Mr. Charles Bonnycastle undertook this task, and was certainly the first to show that the laws which I had deduced from experiment only, were the necessary consequence of a certain hypothesis, which he gave of the nature of magnetic action. There was, however, one point of difference between this gentleman's results and mine, which was merely numerical, and which seemed to be such as required, not any important change in the principles of his investigation, but some slight modification of his first hypothesis.

It followed, for example, from the result of his calculation, that the numerical co-efficient which

* An abstract of this Section is published in the Philosophical Transactions, Part I. for 1822.

I have denoted by A (note to art. 45,) ought to have been about 1-16th greater than I had found it, and moreover that this co-efficient ought to be constant for every species of iron, a result which was not likely to be borne out by experiment, at all events it seemed desirable to decide the question, by making a sufficient number of trials on such varieties of iron and steel, as could be conveniently obtained, and Mr. Bonnycastle being on the spot, we agreed to proceed together in the inquiry: the details of which are given in the following pages:

Experiments on the relative Magnetic Power of different descriptions of Iron and Steel.

127. In these experiments, my first trial was made on four different kinds of metal; viz. cast-iron, soft or malleable iron, soft blistered steel, and hard blistered steel. Of each I had two bars formed, 24 inches in length, and 1¼ inch square. The four bars of blistered steel were of the same quality, except that two of them were softened, and two hardened for the experiments, which latter were conducted in the following manner:

128. The compass was first placed, so as to read correctly north and south; a situation was then taken for the bars, so that the lower end of each, successively, was on a level with the pivot of the compass, and distant from it 10·6 inches, first

to the east, and then to the west: the bar itself being made to incline in the direction of the dip.

For the purpose of recording the results, the ends of each bar were marked, A and B, and the four faces were numbered 1, 2, 3, 4, each of which was successively turned towards the compass; but as little or no difference could thus be detected, I shall simply give the results of each end of the several bars, and the means of each different specimen.

129. *Table of the deviations produced on a Magnetized Needle, by different descriptions of Iron.*

Description of metal.		East Compass.				West Compass.				Mean.	
		End A.		End B.		End A.		End B.			
		°	′	°	′	°	′	°	′	°	′
Cast Iron,	No. 1.	7	30	7	37	8	0	7	45	7	48
Ditto	No. 2.	6	30	9	30	6	0	9	38		
Malleable Iron.	No. 1.	15	30	16	22	15	30	16	22	15	54
	No. 2.	16	0	15	45	16	0	15	45		
Soft blistered steel.	No. 1.	10	56	9	56	10	52	9	56	10	50
	No. 2.	14	22	8	7	14	22	8	7		
Hard blistered steel.	No. 1.	9	56	8	0	10	0	8	0	8	37
	No. 2.	9	30	7	0	9	30	7	0		

A similar series of experiments was made at the distance of 6·7 inches, which gave angles of deviation very nearly proportional to the above.

130. The first thing which calls for particular remark in these experiments is, the near agreement in the mean results of each of the two bars of the same kind, as for example in the two cast iron bars, which differ from each other only a few minutes, and the same with the two of soft iron. With the steel bars there is rather a greater difference, but even here the agreement is sufficiently close to show that the results we have obtained were not merely accidental, but that they exhibit the permanent difference in the magnetic quality of these different kinds of iron.

Another remarkable fact is, that although we find in some of the bars a considerable difference in the action of their two extremities, yet the mean of the two still agree with the mean of the other bar of the same kind. This is particularly exemplified in the two cast iron specimens.

131. I could not procure bars of shear steel of the same lateral dimensions as the above, at least not without the operation of forging, which I was apprehensive might injure the texture, and alter the character of the metal. In order, therefore, to obtain a comparison with the above results, I took four bars of shear steel from the rollers, and had two of them hardened, and two softened for the experiment; I had also two soft iron bars made of the same dimensions; viz. 24-inch long, 1-inch broad, and half an inch thick; these six bars were then

tried precisely in the same manner as in the last experiments. The results were as below:

132. *Preceding Table continued.*

Description of metal.		East Compass. End A.		East Compass. End B.		West Compass. End A.		West Compass. End B.		Means.	
		°	′	°	′	°	′	°	′	°	′
Soft iron.	No. 1.	21	30	23	10	22	0	22	30	22	15
	No. 2.	17	10	27	10	17	20	27	10		
Soft shear steel.	No. 1.	13	20	17	0	not observed.		not observed.		15	0
	No. 2.	12	30	17	10						
Hard shear steel.	No. 1.	10	0	14	0	not observed.		not observed.		12	17
	No. 2.	17	40	7	30						

In these experiments, the distance between the bottom of the bar and the centre of the compass was 5·2 inches.

133. The same remarks apply to these experiments as to the foregoing; viz. that each bar of the same kind of metal, gives very nearly the same mean result, notwithstanding the difference in the action of their extremities.

134. I could only procure one small specimen of cast steel, viz, a bar 9 inches long, and 7-8ths of an inch square: but in order to continue the comparison I had begun, I procured an iron bar of the same dimensions; and tried this latter,

first, against the steel bar, rendered soft, and then against the same when hardened; the following are the results.

135. *Preceding Table continued.*

Description of metal.	East Compass.		West Compass.		Means.
	End A.	End B.	End. A.	End B.	
Soft Iron - -	16° 20′	17° 20′	not	obser.	16° 50′
Cast steel soft -	14 20	11 0	not	obser.	12 40
Cast steel hard -	8 0	8 45	not	obser.	8 22

By collecting the mean results from the three preceding tables, and comparing in each the particular deviations with the corresponding deviation produced by the iron bar; assuming also the tangents of the several angles as the measures of the deflecting power, and that due to the iron bar as unity; we shall have the relative power of the different kinds of metal, as in the last column of the following table:

136. *Table of the proportional Magnetic Powers of different descriptions of Iron and Steel.*

Description of metal.	Angles of deviation		Tangent.	Proportional magnetic power.
	°	′		
Cast Iron - -	7	48	·1369	·479
Malleable iron -	15	54	·2843	1·000
Blistered steel, *soft* -	10	50	·1913	·673
Blistered steel, *hard* -	8	37	·1515	·532
Malleable iron -	22	15	·4091	1·000
Shear steel, *soft* -	15	0	·2679	·655
Shear steel, *hard* -	12	17	·2177	·530
Malleable iron -	16	50	·3025	1·000
Cast steel, *soft* -	12	40	·2247	·743
Cast steel, *hard* -	8	22	·1470	·486

137. Or if we express these ratios by the nearest integral numbers, they will stand thus:

Malleable iron. .	100	Shear steel, hard . .	53
Blistered steel, soft,	67	Cast iron . . .	48
Blistered steel, hard,	53	Cast steel, soft . .	74
Shear steel, soft. .	66	Cast steel, hard. .	49

Experiments on the relative Magnetic Power of Iron and Steel at different degrees of heat.

138. It will have been observed in the detail of the preceding experiments, that the bars of hard

steel exhibited less power on the needle than the soft bars, although the quality in other respects was precisely the same: there could, therefore, be no doubt that the discrepancy in the hard bars was due merely to the process they had undergone, and the impediment which the hardening opposed to the magnetic developement. It became, therefore, a curious question to inquire, how nearly the different species of iron and steel would approximate towards each other, while the metal was soft, by being heated in a furnace.

I determined, therefore, upon a regular series of experiments directed to this inquiry; but, as there appeared to be so little difference between the blistered and shear steel, I confined my experiments only to the latter, and to cast and malleable iron. The smithery, at Woolwich, presented an excellent opportunity of making these experiments upon a proper scale, and I had permission from the honourable the principal Officers and Commissioners of His Majesty's Navy, to avail myself of any such facilities: I accordingly, therefore, with the assistance of Mr. Charles Bonnycastle, undertook the following experiments:

139. Our first essay was made on a cast iron bar about 3 feet long, and 1¼ inch square; which being set up cold in a certain situation selected for the purpose, produced a deviation in the needle, amounting to 21°. It was now put in the furnace

till it had taken a *white heat*, and on being placed in the same situation as before, it was found to have lost all its power on the needle, which settled after a few vibrations due north and south. It continued thus about two or three minutes, when it began to oscillate, and almost immediately reached a deviation of 43°, where it appeared stationary; we now inverted the bar, but the deviation was still 43°; viz. rather more than double what it was when the bar was cold.

140. In the above experiment, the compass was placed near the bottom of the bar, which latter was inclined in the plane of the dip; but we found it more convenient in the next experiment, to place the compass opposite the upper end of the bar, still, however, keeping the latter in the same direction. The piece of iron we now selected was malleable; 25 inches in length, $4\frac{1}{2}$ in breadth, and $\frac{3}{4}$ inch thick. This piece of iron, when cold, gave a mean deviation of 17°; on being brought to a white heat, it first lost all its power on the needle, but as the iron began to assume the colour denoted by the workmen, *blood red heat*, it began to oscillate, and soon attained a deviation of 26°; where it became stationary; and continued the same, which ever of the two ends of the bar was uppermost. We afterwards repeated the experiment with precisely the same results.

These experiments were merely preliminary to

those I had projected, from which it was intended to form a comparison between the magnetic power of malleable, and cast iron and steel.

Comparison of the Magnetic Power of cast and malleable Iron, when heated in a furnace.

141. For the purpose of these experiments, I procured a bar of soft iron, 25 inches in length, and $1\frac{1}{4}$ inch square, and a cast iron bar of the same dimension; but having destroyed the latter by giving it too great a heat in the first experiment, I could only find one of nearly the same size, the length of the new bar was the same, but its side was only $1\frac{3}{10}$ths of an inch.

The compass was placed nearly level with the upper extremity of the bar, and at the distance of 6 inches, the latter being inclined as in the former experiments in the direction of the dip.

The following are the results:

Experiment I.

Cast iron cold.	End A, deviation	21° 30′	mean 21° 30′.
	End B,	21 30	
Ditto, white heat, zero; blood-red			- 62 0.

Experiment II.

Malleable Iron cold.	End A, deviation	37° 0′	mean 40° 0′
	End B,	43 0	
Ditto, white, zero; blood-red			- - 55 0.

These two experiments were repeated with exactly the same results.

The positions of the bars were now changed, viz. they were raised about two inches, but the dip was still observed.

Experiment III.

Cast Iron cold.	End A, deviation	24° 20′	mean 24° 20′
	End B,	24 20	
Ditto,	white heat, zero; blood-red		78 30

Experiment IV.

Malleable Iron cold.	End A, deviation	
	End B, not observed.	
Ditto,	white heat, zero; blood-red	70° 30′

These experiments were repeated with the same results.

142. I should observe here, that the great attraction produced by the heat, did not subside with it, provided the bar remained in its place undisturbed; for after some days I found the power of the bar continue just the same as at the time of making the experiment, when it had not been displaced; but then the bar upon trial was always found to possess a certain degree of fixed magnetism, its other extremity producing an opposite effect upon the needle; but if the bar was inverted, while it retained any visible colour from the heat, both ends produced exactly the same deflection: as to the magnetic effect to which I have alluded above,

it was lost, or at least a great part of it, after leaving the bar for some time horizontal, or, after its being thrown about with other pieces of iron.

143. It is also proper to observe that the needle always begun to indicate the power of the iron upon it, as soon as it arrived at that state of temperature shown by a high blood-red heat, and its motion generally proceeded gradually till it had reached its maximum deviation, which it commonly attained in about a minute or two. It will be remarked as very singular, that cast iron, which is so decidedly inferior in its action, when cold, should possess a superior power when hot, which happened uniformly in every experiment that was made, the two bars being placed under like circumstances: it is moreover to be remembered that the cast iron bar was 1-16th of an inch less in its lateral dimensions, which ought, and necessarily did, diminish its actual power of attraction. Its comparative power is therefore greater than that stated above.

Comparison of the Magnetic Power of soft Iron and shear Steel, when heated in a furnace.

144. The bars employed in these experiments were those, whose effects are recorded in our second table. The results were as follow:

Experiment V.

Malleable Iron cold.	End A, deviation 16° 30′	mean 15° 10′	
	End B, 13 30		
Ditto,	white heat, zero; blood-red -		41 10

Experiment VI.

Soft shear steel, cold.	End A, deviation 11° 30′	mean 11° 0′	
	End B, 10 30		
Ditto,	white heat, zero; blood-red -		48 0.

Experiment VII.

Hard shear steel, cold.	End A, deviation 15° 30′	mean 8° 0′	
	End B, 0 30		
Ditto,	white heat, zero; blood-red -		47 30.

145. The only positive deduction which we are able to make from these experiments is, that soft iron, whose power is so far superior to every other kind of iron and steel when cold, is inferior to any of them when heated; and that cast iron, which has the least power cold, is equal, or superior to any when hot. But there is certainly not so decided a scale of relation in this case, as in the experiments on the cold bars; the principal reason for which may be the damage sustained by the iron by being repeatedly heated.

I should, therefore, now have concluded my experiments on this subject, but for a circumstance which had been noticed, and which strongly attracted our attention. It had been observed both

by Mr. Bonnycastle and myself, in some of the preceding experiments, and others of which I have not given the results, that between the white heat of iron, when its power was actually zero, and the blood-red heat, at which its action manifested itself so highly; there was an intermediate state of the bar, during which it attracted the needle the contrary way to what it did when cold, viz. if the north end of the needle was attracted in the latter state; the south end was attracted, while the heated iron passed through the shades of colour denoted by the workmen the *bright red*, and *red heat*. Our object was, therefore, now to examine this circumstance a little more minutely, than we had hitherto done.

On the Anomalous Attraction of heated Iron, which takes place, while the metal retains the high red, and simple red heat.

146. In our first experiment the compass was placed nearly west of the bar, rather below its upper extremities, and distant from it about 6½ inches. At the white heat, the attraction of the iron was lost; and at the blood-red heat, we had more than 70° of deviation, but that intermediate action we were searching after, did not appear, at least it was by no means so obvious as we had noticed it in our preceding experiments. The po-

sition of the bar and compass, however, was not quite the same as before; we therefore raised the stand for the bar about four inches, by which means its upper extremity was about the same height above the compass; and on repeating the experiment with the bar thus placed, we obtained an obvious deviation of the south end of the needle to the bar of $4\frac{1}{2}°$; which remained fixed for about two minutes.

Having gained this by raising the bar 4 inches, we now raised its base 6 inches; and on applying it in this place, we obtained a deviation of $10\frac{1}{2}°$, which remained fixed about the same time as before; when the needle suddenly yielded to the natural magnetic power of the iron, and obtained almost instantaneously a deviation of 81° the opposite way.

147. It was thus rendered obvious that the quantity of negative attraction at the red heat, depended upon the height or depth of the centre of the bar; and as the natural effect of the cold iron was changed by placing the compass below the centre of the bar, the next question was, will the character of the negative attraction change also? To decide this point; we lowered our compass to within six inches of the bottom of the bar; in which position the cold iron attracted (of course) the south end of the needle, and produced a deviation of 21°; and upon being heated, we found as usual

all its power upon the needle cease at the white heat; but as this subsided into the bright red, the negative attraction begun to manifest itself, and it soon amounted to $10\frac{1}{2}°$: the north end of the needle being attracted towards the iron: it remained stationary for a short interval, and then gradually returned, first to due north, and ultimately to 70° 30′ on the opposite side.

148. I now determined upon making a regular course of experiments, with a view, if possible, to trace this anomalous action to some fixed principle; for it will have been noticed from what is stated above, that the negative attraction appeared to increase from each extremity of the bar towards its middle; whereas the positive or natural action of the iron, decreases in the like cases, and, passing through zero in the plane of no attraction, has its quality of action different when placed towards the upper and lower extremity of the bar. But the negative attraction, which is also different in two opposite halves of the bar, seemed to pass through a maximum to arrive at this change of quality, which appeared wholly inexplicable; and after all the experiments I have made, I must acknowledge that it still remains so. It is at all events certain, that the least change of position, when the compass is opposite the centre of the bar, will change altogether the quantity and quality of this negative action.

149. I should have been glad, in pursuing this inquiry, to have been able to use balls of iron instead of bars, and I made one experiment to see how far it was practicable, but the heat was so intense, that I found it inconvenient, and relinquished this plan for that which I had hitherto pursued.

The result of the ball experiment was as follows: attraction cold + 13° 31′: white heat 0° 0′: red heat — 3° 30′; blood-red heat + 19° 30′.

It may be proper also to observe, that having some doubt, whether the effect I had observed was due to any change in the attractive power of the iron during its change of colour, or whether it might not proceed merely from the heat, I procured two copper bolts, of rather larger dimensions than the iron bars, and had them heated to the highest degree *that* metal would admit of; but upon placing them, when thus heated, in the same situation as the iron bars, no action whatever upon the needle could be detected.

150. In the experiments detailed in the following table, I used four different bars, each 25 inches long, 1¼ inch square, two of them of cast iron, denoted in the first column by C. B. No. 1, C. B. No. 2, and two of malleable iron, denoted by M. B. No. 1; M. B. No. 2; I had also two other bars, one of cast, and one of malleable iron, which were not heated, but kept as standards for

determining the quantity of cold attraction; as this could not be safely done by the bars used in the experiments, after being repeatedly heated in the fire.

The time occupied in each experiment was about a quarter of an hour; the white heat commonly remained about three minutes, when the negative attraction commenced; this lasted about two minutes more, and then the usual attraction begun to indicate its presence; this arrived at its maximum sometimes very rapidly, but at others it proceeded increasing very gradually, and commonly in a quarter of an hour from the beginning the needle had been found perfectly stationary.

N. B. In the following table, to avoid confusion, that attraction which took place according to the usual manner, is marked *plus* +, which ever end of the needle was attracted; and the opposite attraction is marked *minus* —. For example, when the compass is above the centre of the bar, the north end should be drawn towards the bar; but when the compass is below the centre, the south end should be attracted; these, therefore, are both marked + in the table; and the contrary attraction at the red heat is marked —.

151. *Table of Experiments showing the effect of Iron on the Compass Needle at different degrees of heat.*

No. of Experiment.	Description of bar.	Height or depth of centre of bar from compass.	Distance of bar from compass.	Position of the compass.	Effect cold.	Effect white heat.	Effect red heat.	Effect blood-red heat.	Remarks.
		inches.	inches.		° ′	° ′	° ′	° ′	
1	C. B. No. 1.	0	6·0	S. 80° W.	+ 0 0	0 0	— 17 0	0 0	In this experiment the south end was drawn towards the bar at R. H
2	M. B. No. 2.	4·5 below.	6·0	Ditto.	+ 30 0	Ditto.	0 0	+ 45 0	
3	C. B. No. 2.	Ditto.	6·0	Ditto.	+ 18 0	Ditto.	0 0	+ 49 0	
4	M. B. No. 1.	Ditto.	6·0	Ditto.	+ 29 30	Ditto.	— 12 0	+ 44 0	
5	Ditto.	1 3 below.	6·0	Ditto.	not obser.	Ditto.	0 0	+ 52 0	
6	Ditto.	4·5 below.	6·0	N. 80 W.	Ditto.	Ditto.	— 12 30	+ 70 0	This bar being left, it attracted the same three days after.
7	Ditto.	4·5 above.	6·0	S. 80 W.	Ditto.	Ditto.	— 12 30	+ 30 0	
8	Ditto.	Ditto.	6·0	Ditto.	Ditto.	Ditto.	0 0	+ 25 0	The needle was suspected to have touched the bottom of the box.
9	Ditto.	Ditto.	6·0	Ditto.	Ditto.	Ditto.	— 19 0	+ 30 0	
10	Ditto.	1 above.	6·0	Ditto	Ditto.	Ditto.	— 15 0	+ 4 0	
11	M. B. No. 2.	12·5 below.	8·5	N. 80 W.	+ 29 30	Ditto.	0 0	+ 37 30	Observed at the same time with two compasses.
12	Ditto.	Ditto.	8·5	N. 80 E.	+ 30 0	Ditto.	0 0	+ 41 0	
13	C. B. No. 1.	12·5 below.	8·5	N. 80 W.	+ 16 0	Ditto.	0 0	+ 42 30	Ditto ditto.
14	Ditto.	Ditto.	8·5	N. 80 E.	+ 15 30	Ditto.	0 0	+ 47 30	
15	M. B. No. 2.	9·0 below.	8·5	N. 80 W.	+ 28 30	Ditto.	— 1 0	+ 39 30	Ditto ditto.
16	Ditto.	Ditto.	8·5	N. 80 E.	+ 29 30	Ditto.	— 1 30	+ 42 0	
17	C. B. No. 1.	9·0 below.	8·5	N. 80 W.	+ 15 45	Ditto.	— 1 30	+ 45 0	Ditto ditto.
18	Ditto.	Ditto.	8·5	N. 80 E.	+ 16 0	Ditto.	— 1 30	+ 49 0	

TABLE *continued.*

No. of Experiment.	Description of bar.	Height or depth of centre of bar from compass.	Distance of compass.	Position of the compass.	Effect cold.	Effect white heat.	Effect red heat.	Effect blood-red heat.	Remarks.
		inches.	inches.		° ′	° ′	° ′	° ′	
19	M. B. No. 2.	6·0 below.	8·5	N. 80° W.	+ 25 0	0 0	— 3 0	+ 32 30	Observed at the same time with two compasses.
20	Ditto.	Ditto.	8·5	N. 80 E.	+ 26 0	Ditto.	— 3 30	+ 33 0	
21	C. B. No. 1.	6·0 below.	8·5	N. 80 W.	+ 11 30	Ditto.	— 3 30	+ 36 30	Ditto ditto.
22	Ditto.	Ditto.	8·5	N. 80 E.	+ 13 0	Ditto.	not obser.	+ 36 30	
23	M. B. No. 2.	3·0 below.	6·0	S. 80 E.	+ 8 0	Ditto.	— 21 30	not obser.	Ditto ditto.
24	Ditto.	Ditto.	6·0	N. 45 W.	not obser.	Ditto.	— 25 30	+ 25 30	
25	M. B. No. 1.	0 below.	6·0	Ditto.	0 0	Ditto.	— 40 0	0 0	In this experiment the north end was drawn to the iron at the red heat
26	M. B. No. 2.	1·0 above.	5·3	N. 60 W.	+ 2 0	0 0	— 4 30	+ 5 30	Both attractions very gradual.
27	M. B. No. 1.	Ditto.	5·3	Ditto.	not obser.	Ditto.	— 12 30	+ 5 30	Passed suddenly to $12\frac{1}{2}°$, but returned immediately.
28	M. B. No. 2.	9·0 above.	6·0	N. 85 E.	+ 47 30	Ditto.	— 2 30	+ 60 0	Attractions gradual.
29	M. B. No. 1.	Ditto.	6·0	Ditto.	+ 47 30	Ditto.	— 2 30	+ 60 0	Ditto ditto.
30	M. B. No. 2.	1·0 below.	5·5	N. 45 W.	not obser.	Ditto.	— 55 0	+ 5 45	The negative attraction rather sudden.
31	M. B. No. 1.	4·5 above.	7·0	N. 75 E.	Ditto.	Ditto.	— 2 30	+ 33 30	Motion of needle very slow.
32	M. B. No. 2.	1·7 below.	5·5	N. 45 W.	Ditto.	Ditto.	+ 100 0	+ 13 30	The 100° very sudden, returned immediately.
33	M. B. No. 1.	1·7 below.	5·5	Ditto.	Ditto.	Ditto.	— 26 0	+ 13 30	Both attractions gradual.
34	M. B. No. 2.	1·7 above.	5·5	Ditto.	Ditto.	Ditto.	+ 30 0	+ 13 30	The same as in No. 32: both anomalous.
35	M. B. No. 1.	4·5 above.	6·0	N. 55 E.	Ditto.	Ditto.	— 5 30	+ 35 30	Attractions very gradual.
36	M. B. No. 2.	Ditto.	6·0	Ditto.	Ditto.	Ditto.	— 0 0	+ 35 30	No negative attraction.
37	M. B. No. 1.	0·	4·7	West.	+ 3 30	Ditto.	— 50 0	+ 8 0	Motion of needle regular, but quick.
38	M. B. No. 2.	0·	4·7	North.	0 0	Ditto.	0 0	0 0	No motion in the needle.

151. All the above experiments it will be observed, were made with the iron bar inclined in the direction of the dipping needle, or nearly in that direction (for the greatest accuracy was not observed in this respect) and it follows from them, as is stated above, that the negative attraction was the greatest opposite the middle of the bar, where the cold attraction was very small, or zero. I was therefore anxious to ascertain what the effect would be at the red heats, when the bar was placed in a direction perpendicular to the former, or in what I have denominated the plane of no attraction. We accordingly made a few experiments with the bar in this position, but the results were by no means so strongly marked as in the preceding table. We always obtained a certain quantity of negative attraction, as in the former cases, but its amount was very inconsiderable; in no instance exceeding $2\frac{1}{2}^{\circ}$.

The only explanation which seems to present itself of the cause of this anomalous action, is, that the bar cooling faster at its extremities than in its centre, one part of it becomes magnetic before the other, and hence gives rise to the irregular action above indicated. It must be acknowledged, however, that this explanation does not meet entirely all the phenomena recorded in the preceding table.

PART II.

Containing a Theoretical Investigation of the Laws of Induced and Terrestrial Magnetism.

SECTION I.

INVESTIGATION OF THE LAWS OF MAGNETISM PECULIAR TO IRON BODIES.

152. It has been already stated that soon after my Essay was published, Mr. C. Bonnycastle undertook to deduce the several laws arising out of the experiments, from a theory, exceedingly simple in itself, founded on a supposed similarity of action between electrified and magnetized bodies, and employing accordingly the principles laid down by Poisson in the volume of the Institute for 1811, for establishing the laws of action in the former class of bodies.

But although this ingenious investigation explained, to a certain extent, the laws and deductions arising out of my experiments, yet it led to a consequence which appeared somewhat improbable; viz. that every species of iron possesses the same degree, or rather perhaps admits of the same degree, of magnetic developement while the same intensity of action resides in the disturbing body; that is, a ball of cast iron, malleable iron, and of

steel, of the same dimension, would produce the same quantity of deviation in a magnetized needle submitted to its influence.

This result led to the series of experiments detailed in the preceding section, and with which it is obviously irreconcilable. Some modification therefore became requisite, and I have adopted the following, which is independent of Poisson's principles; these being apparently inapplicable to magnetized bodies; although it is remarkable that Mr. Bonnycastle succeeded, by founding his investigation upon them, in establishing theoretically every result which I had obtained, with only one exception; viz. the numerical value of the co-efficient deduced, in (art. 43) which co-efficient, as already observed, ought, according to the principles adopted, to be constant for every species of iron.

153. The paper above alluded to was published in the Philosophical Magazine, vol. lv. p. 132, with a supplementary article at p. 446 of the same volume. The following investigation is founded on a modification of the hypothesis there adopted, and the investigation is thrown into a different form, but still the leading principles of the solution are the same.

154. The hypothesis upon which I shall proceed, may be thus enunciated :—

1. Magnetic phenomena are due to the existence

of two fluids in a greater or less degree of combination, and such, that the particles of the same fluid repel, and those of an opposite nature attract, each other.

2. These fluids in iron bodies exist naturally in a state of combination and equilibrium, till that state is disturbed by some exciting cause.

3. But if a body, already magnetic, *i. e.* one in which these fluids are held in a state of separation, be brought within the vicinity of a mass of iron, such as is supposed above, the concentrated action of each fluid in the magnetized body will act upon the latent fluids in the quiescent body, by repelling those of the same, and attracting those of the contrary kind, and thus impress upon the latter a temporary state of magnetic action, which will remain only while the two bodies maintain their respective situations.

4. The quantity of action thus impressed upon the iron body will depend, *first*, upon the intensity of the exciting magnet; *secondly*, upon the capacity of the quiescent body for magnetism, or the quantity of those fluids contained in it, and *thirdly*, upon the cohesive power of the iron; which latter quality determines the depth to which the exciting magnet is able to disengage the two fluids.

The above embraces every case; viz. of any magnet, natural or artificial, developing the magnetism in any given iron body; but in that to

which our attention will be principally directed, namely, the displacement occasioned by the magnetic action of the earth on spheres of iron, we shall find more limited in its results, and more susceptible of correct mathematical investigation.

5. In this case, for instance, we may suppose the action to take place on every particle of the mass in lines parallel to each other, and corresponding with the direction of the dipping needle; also that every particle is at the same distance from the centre of the disturbing force, and consequently that the displacement in each particle is equal also; conditions which throw great facilities into the analytical investigation of the laws of action.

6. For the sake of illustration, let A B C D (fig. 11.) represent a sphere of iron in its non-magnetic, or quiescent state, and let C M be the line in which the terrestrial magnetism is exerted from a centre of action, M, which is at such a distance that the diameter of the sphere is inconsiderable in comparison with it; then every particle on its surface, and to a certain distance within it, will be acted upon by equal powers, and in directions parallel to each other; whereby the fluids in the quiescent body, before in a state of combination, will be separated in each particle; and the two fluids may now, therefore, be conceived to form two spherical shells, A *e* B *d*, A *e'* B *d'*, whose centre of action

will be in c, c', their distance from each other being greater or less, according to the circumstances stated in (No. 4).

7. Therefore in computing the action of such a mass of iron in its temporary state of magnetism upon a distant particle of magnetic fluid, we may refer it to those centres; we shall also assume, that the law of action in this, as in all other cases of central action, is inversely as the square of the distance.

155. Such is the hypothesis upon which is founded the following investigation, and which, it will be seen, will enable us to deduce all those laws previously drawn from experiment, as likewise to infer several curious consequences arising out of our analytical formulæ; the probable accuracy of which will be estimated by the coincidence in those cases between theory and experiment, where the comparison can be made.

156. It may be observed that this hypothesis differs from that advanced by Mr. Bonnycastle only in this; *i. e.* he imagined the fluids to be separated from each other and to be accumulated in distinct poles, or centres of action; whereas, according to the supposition made above, the displacement takes place in each particle, and the centres of action are, therefore, indefinitely near to each other in the common centre of attraction of the surface of the body. It differs also from

the theory advanced by Coulomb, in this, that he conceives the displacement to take place in every particle of the mass; whereas we suppose it to be confined to the surface, a fact which has been already established by experiment, (art. 47 et seq). His centres of action are, therefore, in the centre of attraction of the mass, and our's in the centre of attraction of the surface. These are coincident in spheres, but in no other bodies.

157. Agreeably to the hypothesis which has been advanced, let A B (fig. 12.) represent a sphere of soft iron, which has acquired a magnetic action in the direction S N, from the effect of the terrestrial magnetism; S N denoting the line of the dip; and let P be an indefinitely small magnetic particle. Then this, by the supposition, will be acted upon, first by the terrestrial magnetism in a direction parallel to S N, and also by the sphere A B from the two centres c, c' indefinitely near to each other, and to the geometrical centre of the sphere o; being repelled from one of those centres and attracted towards the other, by forces varying inversely as $P\,c^2$ and $P\,c'^2$; and in consequence of these forces, the particle at P will assume a certain direction, which it is our object to compute.

158. Let c' be the centre towards which the particle is attracted, and c that from which it is repelled; and let the effect due to the former be

denoted by $\frac{f}{P c'^2}$, then that due to the latter will be $\frac{f}{P c^2}$. In the same way, if we consider the action on the other pole of the particle at P, we shall find the repulsion upon it denoted by $\frac{f}{P c'^2}$, and the attraction by $\frac{f}{P c^2}$, which will produce precisely the same results as the former forces; we may, therefore, consider each of these forces as being doubled, and to become $\frac{2f}{P c'^2}$ and $\frac{2f}{P c^2}$.

Let the distance Po be denoted by d, the angle S o P by ϕ, and the indefinitely short distance $c\,o = c'o$ by e.

Conceive the force $\frac{2f}{P c^2}$ to be resolved into two forces, in the direction o P, $c\,o$, and the force $\frac{2f}{P c'^2}$ into two, in the directions P o, $o\,c'$.

That is, resolve

$$\frac{2f}{P c^2} \text{ into } \frac{2fd}{P c^3} \text{ and } \frac{2fe}{P c^3}$$

$$\frac{2f}{P c'^2} \text{ into } \frac{2fd}{P c'^3} \text{ and } \frac{2fe}{P c'^3}$$

159. The first of each of these pairs of forces are opposed to each other's action, and their resultant is therefore equal to their difference; but the others acting in the same direction, $c\,c'$ must be added; our forces thus become

$$\frac{2fd}{P c'^3} - \frac{2fd}{P c^3} \text{ in the direction P } o,$$

and $\frac{2fe}{\mathrm{P}c'^3} + \frac{2fe}{\mathrm{P}c^3}$ in the line $c\,c'$ or N S, the latter being opposed to that of the terrestrial action.

Now, by trigonometry

$$\mathrm{P}c = \sqrt{(d^2 + e^2 + 2\,d\,e\cos\phi)}$$
$$\mathrm{P}c' = \sqrt{(d^2 + e^2 - 2\,d\,e\cos\phi)}$$

The above expressions, therefore, become

$$\frac{2fd}{(d^2 + e^2 - 2\,d\,e\cos\phi)^{\frac{3}{2}}} - \frac{2fd}{(d^2 + e^2 + 2\,d\,e\cos\phi)^{\frac{3}{2}}} \text{ in P}o$$

$$\frac{2fe}{(d^2 + e^2 - 2\,d\,e\cos\phi)^{\frac{3}{2}}} + \frac{2fe}{(d^2 + e^2 + 2\,d\,e\cos\phi)^{\frac{3}{2}}} \text{ in N S}$$

But since e is indefinitely small in respect to d, we may omit in the developement of these formulæ all those terms in which e enters in any power higher than the first, and which thus reduce to

$$\frac{12fe\cos\phi}{d^3} = \text{force in P}o \ldots (1)$$

$$\frac{4fe}{d^3} = \text{force in N S} \ldots (2)$$

The forces, therefore, arising from the action of the sphere upon the particle P are resolved into two, which may be represented by the lines Pm, Pn; and consequently their resultant will fall somewhere within the angle m Pn. Let us, however, before we attempt this composition, introduce the natural directive power of the earth upon the needle, and ascertain the proper analytical value of the force denoted by f.

160. With respect to the former, since it is constant in the line S N, it will be sufficient to

denote it by any constant quantity M; and for the latter, we may observe, that agreeably to our hypothesis, (art. 157) it follows that in iron of the same species, and of a thickness greater than the depth of metal at which the developement is effective, the quantity of displaced magnetism will be directly as the surface, or as the square of the radius; but its central action upon a particle at the surface will be inversely as the square of the distance, or square of the radius; consequently, the action of those centres on a particle at the surface will be the same for spheres and shells of all diameters and dimensions: hence our force (2) at the surface, which there becomes $\frac{4fe}{r^3}$, is a constant quantity, whatever may be the value of r: make it equal to C, and we shall have

$$\frac{4fe}{r^3} = \text{C, or}$$

$$f = \frac{\text{C}\,r^3}{4\,e}$$

Substituting this value of f, into our forces (1) and (2), and combining with the latter the constant directive force M, we shall have for the forces acting on the particle P,

$$\frac{3\,\text{C}\,r^3 \cos\phi}{d^3} = \text{force in P}\,o \qquad (3)$$

$$\text{M} - \frac{\text{C}\,r^3}{d^3} = \text{force in S N} \qquad (4)$$

For the more convenient estimation of these

forces, let the former be resolved into two, the one perpendicular to S N, and the other parallel to it; by which means they will become

$$\frac{3\,C\,r^3 \cos\phi \sin\phi}{d^3} = \text{force perp. to S N} \qquad (5)$$

$$M + \frac{3\,C\,r^3 \cos^2\phi}{d^3} - \frac{C\,r^3}{d^3} = \text{force in S N} \qquad (6)$$

Let Δ represent the angle which the resultant of these forces forms with N′ S′, or P N′, then, by the principle of forces, we shall have

$$\tan\Delta = \frac{3\,C\,r^3 \cos\phi \sin\phi}{M\,d^3 + 3\,C\,r^3 \cos^2\phi - C\,r^3}$$

or

$$\tan\Delta = \frac{3 \cos\phi \sin\phi}{\frac{M\,d^3}{C\,r^3} + 3\cos^2\phi - 1} \qquad (7)$$

162. In this expression, Δ obviously denotes the deviation of the magnetic particle from its natural direction S N, or of an indefinitely short needle freely suspended at that point. This deviation being, therefore, that due to a needle freely suspended, will necessarily take place in the plane which passes through the centre of the needle and the two centres of the attracting body. To estimate the effect of this force in deflecting a horizontal needle, the two forces (5) and (6) may be resolved into two, perpendicular and parallel to the meridian; and to estimate the same on a dipping needle suspended in the plane of the meridian, they may in like manner be resolved into two, the one perpendicular, and the other parallel, to the horizon.

Or, which is perhaps more simple, we may project the arc Δ (7) in the plane above alluded to, upon the horizontal plane, and on the plane of the meridian, and thus determine the deviations required.

Of the Horizontal Needle.

163. First, for the horizontal needle; let S N (fig. 14) represent the needle in its natural direction, about which and concentric with it, conceive a sphere to be described. Let P denote the centre of the attracting body, and S S′ the arc of deviation in the plane S S′ o P, (which has been denoted by Δ) H H′ the horizon, Q Q the equator, or plane of no attraction; and let the angle which the plane S o P makes with the meridian be denoted by i, that is, make $\angle$ Z S S′ $= i$, (Z being the zenith,) the angle S o P $= \phi$ as before, and the arc H S $= \delta$, the dip of the needle.

If now from the zenith Z, we draw the arc Z S V, H V will be the projection of the arc S S′ upon the horizontal plane, and will exhibit the angle of deviation in a needle limited in its motion to that plane.

Make this last angle $= \Delta'$, then by a trigonometrical formula, readily deduced, we have

$$\tan \Delta' = \frac{\tan \Delta' \sin i}{\cos \delta - \dfrac{\sin \delta \cos i}{\cot \Delta}} \qquad (8)$$

Into which introducing the value of tan Δ, already determined; viz.

$$\tan \Delta = \frac{3 \cos \phi \sin \phi}{\frac{M}{C}\frac{d^3}{r^3} + 3 \cos^2 \phi - 1}$$

we find

$$\tan \Delta' = \frac{3 \cos \phi \sin \phi \sin i}{\frac{M}{C} \cdot \frac{d^3}{r^3} \cos \delta + (3 \cos^2 \phi - 1) \cos \delta - 3 \sin \phi \cos \phi \sin \delta \cos i} \quad (8)$$

Or, which is the same,

$$\tan \Delta' = \frac{\frac{3}{2} \sin 2 \lambda \cos l}{\frac{M}{C} \cdot \frac{d^3}{r^3} \cos \delta + (3 \sin^2 \lambda - 1) \cos \delta \mp \frac{3}{2} \sin 2 \lambda \sin \delta \sin l}$$

Or making

$$(3 \sin^2 \lambda - 1) \cos \delta \mp \frac{3}{2} \sin 2 \lambda \sin \delta \sin l = N$$

(the lower sign applying to the case of $i > 90°$)

we have more concisely

$$\tan \Delta' = \frac{\sin 2 \lambda \cos l}{\frac{M}{C} \cdot \frac{2 d^3}{3 r^3} \cos \delta + \frac{2}{3} N} \quad (9)$$

where λ denotes the complement of the angle ϕ, or the latitude, and l the complement of the angle i, or the longitude, conformably with the notation employed in the preceding sections of this Essay.

164. This is a rigorous formula for all distances, and for a ball or shell of any radius, and of any description of iron; but the value of the co-efficient $\frac{M}{C}$ is different for different species of iron. In the

case of cast iron, it will be seen in the following article, that it is very nearly equal to unity, and consequently in this case, when d exceeds two or three times r, as in my experiments; the quantity denoted by N, which is made up with the sums and differences of products, of which each of the trigonometrical factors is less than unity, is small in comparison with the other term $\frac{M}{C} \cdot \frac{2d^3}{3r^3} \cos \delta$ and we shall, therefore, by rejecting the former, have as an approximate formula,

$$\tan \Delta' = \frac{C}{M} \cdot \frac{3r^3}{2d^3 \cos \delta} (2 \lambda \cos l)$$

which exhibits in one term, all the approximative laws deduced from the experiments; that is, it follows from this formula,

1. That although by the hypothesis the developement of the magnetism of the ball or shell takes place only at its surface, yet the effect of it as shown by the tangent of deviation is proportional to the cube of the radius or diameter.

2. That the tangent of deviation is inversely as the cube of the distance.

3. That the tangent of deviation is proportional to the sine of the double latitude and cosine of the longitude, the latter being estimated from the magnetic east and west points, which are precisely the laws deduced from the experiments.

165. With reference to the numerical value of

the constant co-efficient $\frac{C}{M} \cdot \frac{3\,r^3}{2\,d^3} \cos \delta$ it may be found by comparing it with the experimental value of the same as deduced from our experimental results, given in arts. 23, 24, 25, 26, or rather from the two latter; because the distance in these are greater than in the two former, and it is obvious from our formula, that the approximation will be nearer as the distance is greater, the rejected term N being in that case less considerable in respect to the constant part.

Hence, since

$$\tan \Delta' = \frac{C}{M} \cdot \frac{3\,r^3}{2\,d^3 \cos \delta} (\sin 2\,\lambda \cos l)$$

we have, when $\cos l = 1$, as in the experiments in question,

$$\frac{\sin 2\,\lambda}{\tan \Delta'} = \frac{M}{C} \cdot \frac{2\,d^3 \cos \delta}{3\,r^3}$$

The exact diameter of the ball is 12·8 inches, or radius 6·4 inches, and in one case the distance is 18 inches, and in the other 20 inches. Hence our tabular numbers give,

$$1^{st} \ldots\ldots \frac{M}{C} \cdot \frac{2.\,18^3.\,\cos 70^\circ\tfrac{1}{2}}{3\,.\,\overline{6{\cdot}4}^3} = 4{\cdot}709$$

$$2 \ldots\ldots \frac{M}{C} \cdot \frac{2.\,20^3.\,\cos 70^\circ\tfrac{1}{2}}{3\,.\,\overline{6{\cdot}4}^3} = 6{\cdot}432$$

The first of these gives $\frac{C}{M} = 1{\cdot}0558$,

and the second $\frac{C}{M} = 1{\cdot}0513$

Mean $\frac{C}{M} = 1{\cdot}0535$

This number is constant for cast iron balls and shells of every diameter, and for all distances and positions; our correct formula, (9) therefore, becomes

$$\tan \Delta' = \frac{\sin 2\lambda \cos l}{\frac{2}{3}\left\{\frac{d^3}{1{\cdot}0535\, r^3} + 3\sin^2\lambda - 1\right\}\cos\delta \mp \sin 2\lambda \sin\delta \sin l}$$

or

$$\cot \Delta' = \frac{2}{3\sin 2\lambda}\left\{\frac{d^3}{1{\cdot}0535\, r^3} + 3\sin^2\lambda - 1\right\}\cos\delta \operatorname{see} l \mp \sin\delta \tan l$$

(10) which is the most convenient form for computation.

166. As this formula is wholly theoretical, with the exception of the numerical co-efficient $\frac{1}{1{\cdot}0535}$, it will be satisfactory to compare the results deduced from it with those obtained from actual experiment, and we have an excellent opportunity of doing this by means of the table of experiments published by Mr. Christie in the first part of the Transactions of the Cambridge Philosophical Society. These were made by that gentleman on the same apparatus which I had employed, but in different circles; namely, in parallels of latitude corresponding to 30°, 45°, and 60°, and at every 10th degree of longitude. They were made with a new and accurate compass constructed for the purpose, and every precaution was used to assure the greatest possible accuracy in the results.

The distance in all these experiments was 18 inches, or $d = 18$; the radius of the ball was 6·4 inches, or $r = 6{\cdot}4$; and dip $\delta = 70° 30'$; whence the constant part of the above formula for each different parallel of latitude are, for

lat. 30° - - $\cot \Delta' = 5{\cdot}360 \sec l \mp \tan l \sin 70°\frac{1}{2}$
lat. 45° - - $\cot \Delta' = 4{\cdot}809 \sec l \mp \tan l \sin 70°\frac{1}{2}$
lat. 60° - - $\cot \Delta' = 5{\cdot}745 \sec l \mp \tan l \sin 70°\frac{1}{2}$

Table containing a comparison between Mr. Christie's *periments, and the theoretical results computed by the preceding rmula.*

Latitude 30°			Longitude.	Latitude 45°			Longitude.	Latitude 60°		
Computed.	Observed	Error.		Computed.	Observed	Error.		Computed.	Observed	Error.
° ′	° ′		°	° ′	° ′		°	° ′	° ′	
2 14	2 20	— 6	80 N	2 34	2 33	+ 1	80 N	2 4	2 5	— 1
4 22	4 44	— 22	70	5 2	5 0	+ 2	70	4 1	3 58	+ 3
6 17	6 52	— 35	60	7 9	7 12	— 3	60	5 48	5 43	+ 5
7 53	8 16	— 23	50	8 57	8 53	+ 4	50	7 18	7 29	— 11
9 9	9 21	— 12	40	10 20	10 1	+ 19	40	8 29	8 26	+ 3
0 2	9 58	+ 4	30	11 18	10 54	+ 24	30	9 19	9 5	+ 14
0 34	10 23	+ 11	20	11 50	11 29	+ 21	20	9 50	10 1	— 11
0 44	10 38	+ 6	10	11 58	11 45	+ 13	10	10 1	9 46	+ 15
0 34	10 17	+ 17	0	11 45	11 32	+ 13	0	9 52	9 43	+ 9
0 7	9 56	+ 11	10 S	11 13	11 8	+ 5	10 S	9 27	9 21	+ 6
9 23	9 8	+ 15	20	10 22	10 16	+ 6	20	8 48	8 38	+ 10
8 27	8 16	+ 11	30	9 19	9 17	+ 2	30	7 56	7 49	+ 7
7 19	7 3	+ 16	40	8 3	7 58	+ 5	40	6 53	6 45	+ 8
6 2	6 3	— 1	50	6 38	6 37	+ 1	50	5 41	5 35	+ 6
4 38	4 38	0	60	5 5	5 0	+ 5	60	4 21	5 25	— 4
3 8	3 9	— 1	70	3 27	3 20	+ 7	70	2 58	2 54	+ 4
1 35	1 38	— 3	80	1 44	1 40	+ 4	80	1 29	1 30	— 1

168. It would be useless to expect a closer approximation between theory and practice, in experiments of such a nature, which, notwithstanding all the care that can be used, are subject to errors which it is impossible to estimate; the daily variation alone, for example, may cause an error of 15′, and very few of our errors in the preceding Table exceed this amount; besides which, we have to allow for any trifling deviation in placing the meridian line on the table in the plane of the actual meridian, in the adjustment of the compass at each place of observation, and in measuring its distance from the centre of the table and its depth below the centre of the ball. When these sources of error are properly considered, it will be found that the agreements between the theory and experiment is as close as can be reasonably expected. It is to be farther observed, that we have assumed, for the sake of simplifying the computation, that the needle is indefinitely small, whereas it must necessarily be of a determinate length; but while this is less than one third of the distance, the errors thence arising, although of some amount, are very inconsiderable.

Of the Dipping Needle.

169. Having completed our computation on the horizontal needle, let us next inquire what the

effect will be upon a dipping needle limited in its motion to the meridian. For this purpose we have only to project the arc Δ, determined in formula (7), upon the plane of the meridian, by drawing the arc S′ E (fig. 14) perpendicular to the circle H Z H′; so will S E be the deviation sought.

Since S′ E S is a right angled triangle, we have immediately,

$$\tan S E = \tan S S' \cos i.$$

But $S S' = \Delta$, and by formula (7)

$$\tan \Delta = \frac{3 \sin \phi \cos \phi}{\frac{M}{C} \cdot \frac{d^3}{r^3} + 3 \cos^2 \phi - 1}$$

Whence denoting the arc S E by Δ'' we have

$$\tan \Delta'' = \frac{3 \sin \phi \cos \phi \cos i}{\frac{M}{C} \cdot \frac{d^3}{r^3} + 3 \cos^2 \phi - 1} \qquad (10)$$

from which the angle Δ'' may in any case be computed.

But for the purposes of computation it will be best to convert the above expression into

$$\cot \Delta'' = \frac{\frac{M}{C} \frac{d^3}{r^3} + 3 \cos^2 \phi - 1}{3 \sin \phi \cos \phi \cos i}, \text{ or}$$

$$\cot \Delta'' = \left\{ \left(\frac{M}{C} \frac{2 d^3}{3 r^3} - \frac{2}{3} \right) \operatorname{cosec} 2 \lambda + \tan \lambda \right\} \operatorname{cosec} l$$

in which it is only requisite to introduce the proper value of the several quantities, as determined in (art. 165).

Comparison of this formula with the results of experiments on the Dipping Needle.

170. The experiments, of which the results are given in the following Table, were made on the same apparatus as that employed in my former experiments. The needle was placed only in the plane of the meridian, at such distances from the centre of the table, and the ball elevated so much above the centre of the needle, as to bring it into the several latitudes $7^\circ\frac{1}{2}$, 15°, $22^\circ\frac{1}{2}$, &c. keeping, in all the experiments, the centre of the ball and centre of the needle at the constant distance of 20 inches from each other.

We have, therefore, $\frac{M}{C} = \frac{1}{1{\cdot}5035}$ (art 165) $r = 4{\cdot}6$ inches, $d = 20$ inches, and cosec $l = 1$.

171. Hence the above formula in numbers becomes

$$\cot \Delta'' = 18{\cdot}639 \operatorname{cosec} 2\lambda + \tan \lambda$$

from which the column of computed deviations, in the first of the annexed Tables, has been calculated.

172. If we take $r = 8{\cdot}85$, the radius of an 18 inch shell, then the above becomes

$$\cot \Delta'' = 6{\cdot}6341 \operatorname{cosec} 2\lambda + \tan \lambda$$

which is the formula employed in Table 2.

173. *Table showing the deviations produced in a Dipping Needle by an Iron Ball* 13 *inches in diameter; and the computed deviations from the preceding formula. Distance* 20 *inches, mean detached dip* 70° 40′.

Latitude.		Observed Dip.		Observed deviation.		Computed deviation.	
°	′	°	′	°	′	°	′
0	0	70	40	0	0	0	0
7	30	69	40	1	0	0	48
15	0	68	55	1	45	1	31
22	30	68	20	2	20	2	9
30	0	68	10	2	30	2	36
37	30	67	40	3	0	2	51
45	0	67	40	3	0	3	4
52	30	67	55	2	45	2	41
60	0	68	10	2	30	2	24
67	30	68	40	2	0	1	58
75	0	69	10	1	30	1	23
82	30	69	40	1	0	0	43
90	0	70	55	0	15	0	0

The above mean dip 70° 40′, was with one end of the needle only, without inverting the poles: the error of the instrument is not included.

174. *Table showing the deviations produced in a Dipping Needle by an* 18 *inch Shell, and the computed deviations from the preceding formula. Distance* 20 *inches, mean detached dip* 70° 11′.

Latitude.		Observed Dip.		Observed deviation.		Computed deviation.		Time of making 40 vibrations.
°	′	°	′	°	′	°	′	″
0	0	70	11	0	0	0	0	125·3
15	0	66	7	4	4	4	14	124·44
30	0	63	49	6	22	6	56	121·3
33	30							120·3
45	0	62	43	7	28	7	28	116·5
60	0	64	22	5	49	6	5	113·58
75	0	65	44	3	27	3	21	112·3
90	0	70	11	0	0	0	0	110·2

The above experiments were made with a different instrument from the former; the needle was on Captain Kater's construction, viz. diamond formed; 7 inches in length, the action of the needle remarkably correct. The above is the dip with one end only.

175. The last column shows the number of seconds in which the needle made 40 vibrations in each position; the object was to compare the computed with the observed intensity, as will be explained in a subsequent article. These observations were made with great care, the time being counted by a machine which would register to 40ths of seconds. The face of the instrument

was first turned to the East and then to the West, the dip and vibrations being registered four times in each position, and the mean of the eight results taken, as they are given in the foregoing table.

176. Confining ourselves here only to notice the deviations, we think it must be admitted that the agreement is as near as can be reasonably expected; both the instruments employed were perhaps amongst the most perfect of their kind; but every one who is acquainted with the dipping needle will be aware that it is not so constant in its action as we might desire; the difference between any two dips, however, in the same position, never exceeded half a degree; and as to the number of vibrations, they seldom differed from each other by half a second.

General Results.

177. If we now collect under one point of view our principal formula, (7), (9) and (10), and substitute in them the proper values in λ and l, instead of their complements ϕ and i, we shall have, after the requisite reductions,

$$\tan \Delta = \frac{\sin 2\lambda}{\frac{M}{C}\,\frac{2\,d^3}{3\,r^3} + 2\sin^2\lambda - \frac{2}{3}} \qquad (11)$$

$$\tan \Delta' = \frac{\sin 2\lambda \cos l}{\frac{M}{C}\cdot\frac{2\,d^3}{3\,r^3}\cos\delta + \frac{2}{3}N} \qquad (12)$$

$$\tan \Delta'' = \frac{\sin 2\lambda \sin l}{\frac{M}{C} \cdot \frac{2\,d^3}{3\,r^3} + 2\sin^2\lambda - \frac{2}{3}} \qquad (13)$$

178. These are all rigorous formulæ; the first shows the deviation of a needle freely suspended, or free to move in every direction; the second, the deviation of a needle limited in its motion to the plane of the horizon; and the third, the deviation caused in the dipping needle placed in the plane of the meridian. And if, in these formulæ, we reject all the terms in their respective denominators beyond the first, for the reasons assigned (art. 164) we shall have the following simple approximate formulæ, answering to these respective cases, viz.

$$\tan \Delta = \frac{C}{M}\frac{3\,r^3}{2\,d^3} \sin 2\lambda \qquad (14)$$

$$\tan \Delta' = \frac{C}{M}\frac{3\,r^3}{2\,d^3} \sin 2\lambda \cos l \sec \delta \qquad (15)$$

$$\tan \Delta'' = \frac{C}{M}\frac{3\,r^3}{2\,d^3} \sin 2\lambda \sin l \qquad (16)$$

179. From these formulæ, or from the three preceding ones, we may draw several curious results, viz.

1. It is obvious, that in all the above formulæ, when $\lambda = o$, the deviation will be zero, which answers to the case of the body being placed in the plane of no attraction.

2. In the second formula, we shall also have $\Delta' = o$, when cos $= o$, or when l, the longitude is 90°, which answers to the plane of the meridian.

3. Again, in the third, $\Delta'' = o$, when $\sin l = o$, or when the longitude is zero; which answers to the plane cutting the equator or plane of no attraction, at right angles, and passing through the east and west points of the horizon.

There are therefore three planes of no attraction, of which however one only is general for all cases, the two others are partial, one belonging to the horizontal, and the other to the dipping, needle, as above stated.

180. We may draw similar conclusions in reference to the maximum of deviation, but they would be somewhat complicated if we deduced them from our correct formulæ (11), (12), (13); they are, however, equally as simple as the above if we confine ourselves to the approximate formulæ (14), (15), (16), which, although they are not strictly correct, will give results very nearly true. From these it appears,

1. That every thing being supposed constant but the latitude, the deviation will be the greatest when $\sin 2\lambda$ is the greatest, that is when λ or the latitude $= 45°$. This is common to all the three cases.

2. Every thing being constant but the longitude, the deviation will be the greatest in the horizontal needle, when $\cos l$ is the greatest, that is when $\cos l = 1$, or $l = o$.

3. The same being supposed as above, the

deviation will be the greatest in the dipping needle, when $\sin l$ is the greatest, or when $\sin l = 1$, or $l = 90°$.

Hence the deviation in the dipping needle will be the greatest where the horizontal deviation is nothing, and the deviation in the horizontal needle the greatest where that of the dipping needle is nothing. Every thing but the longitude being supposed constant.

4. We may also compare the actual quantity of the deviations in the two needles under the same circumstances of latitude, longitude, &c.; for it is obvious that

$$\tan \Delta' : \tan \Delta'' :: \cos l \sec \delta : \sin l, \text{ or}$$
$$\tan \Delta' : \tan \Delta'' :: 1 : \tan l \cos \delta$$

When $l = 45°$, the last analogy becomes

$$\tan \Delta' : \tan \Delta'' :: 1 : \cos \delta$$

which is also the ratio of the respective maximum values of those angles; for from what is stated above, these will be

$$\tan \Delta' : \tan \Delta'' : \sec \delta : 1 :: 1 : \cos \delta$$

5. As the $\cos \delta$, in these latitudes, is nearly equal to $\frac{1}{3}$; it follows, that the greatest deviation caused in a dipping needle, in England, by the action of a mass of iron will be but one third of the greatest effect which the same mass will produce in a horizontal needle; the distance in both cases being the same.

6. But when $\cos \delta = 1$; that is, when the dip is

zero; then the deviations of both needles at their maximum, and in the longitude 45°, will be equal to each other.

181. Another remarkable deduction from our formulæ (15), is, that while the compass is placed in the horizontal circle passing through the centre of the ball, the amount of deviation is independent of the dip; or is the same in all parts of the world. For let m (fig. 15) be the place of the compass in the horizontal circle H H′, then its latitude λ will be denoted by the arc $m\,n$, and its longitude l, by Wn, also the angle $m\,Wn$ will be the complement of the dip.

Hence in the right angled triangle $m\,Wn$ we have

$$\sin Wm = \sin m\,n \operatorname{cosec} m\,Wn = \sin\lambda \sec\delta$$
$$\cos Wm = \cos m\,n \cos Wn \quad = \cos\lambda \cos l$$

Therefore,

$$\sin . Wm . \cos Wm = \sin\lambda \cos\lambda \cos l . \sec\delta$$

or,

$$\sin 2\,Wm = \sin 2\lambda \cos l \sec\delta$$

Hence, in the particular case in question, our formula (15) becomes, (by making $Wm = h$)

$$\tan\Delta' = \frac{C}{M}\,\frac{3\,r^3}{2\,d^3}\sin 2\,h$$

where h is simply the arc subtended between the compass and the east or west points of the horizon.

When the compass, therefore, is thus situated, the deviation is the same at the same points in all

parts of the world, the radius r and distance d being constant.

Mr. Charles Bonnycastle, who first made this deduction, proposes to employ it for determining the local attraction of a ship in all parts of the world; that is, we must determine by experiment, that place in the vessel which is in the same horizontal line with the centre of attraction of all the iron; then a compass there placed would have a constant deviation at each point, and consequently might be used as a standard of comparison for the other compasses.

182. This, however, is on the supposition that the ratio of C to M is a constant quantity, which may not perhaps be strictly true. According to our hypothesis, C, which denotes the power developed at the surface of the sphere, is greater or less according to the power of the exciting magnet, and therefore ought to vary as M varies, the latter representing the natural magnetic intensity at any place; and that this change actually takes place there can be no doubt; but we are not so certain that the value of C is exactly proportional to M, because the resisting power of the iron is the same in all cases, and therefore opposes in all, an equal resistance; and the co-efficient C having reference to the resistance to developement as well as to the exciting power, may not be exactly proportional to M. This, however, is a question which can

only be satisfactorily answered by appropriate experiments in different parts of the earth.

182. Should this equality of ratio between C and M be ultimately established, then it will follow from our formula (15), that under like circumstances of mass, distance, and magnetic position, the deviation in a horizontal needle will vary as the secant of the dip; but that in the dipping needle the deviation will be constant; the formula (16) expressing this deviation having reference only to mass, distance, and position.

SECTION II.

ON THE CHANGE IN THE MAGNETIC INTENSITY OF A NEEDLE AS AFFECTED BY IRON SPHERES.

183. In the preceding investigations our object has been merely to compute the deviation caused in a horizontal or dipping needle by the magnetic action of the iron sphere; that is, we have had only to compute the angle which the resultant of our two forces (5) (6) makes with the natural direction of the magnetic force; our object now will be to find the actual value of that resultant.

This, from the known principles of mechanics, will be, referring to the above formulæ,

$$\sqrt{\left\{\left(\frac{3C\,r^3}{d^3}\cos\phi\sin\phi\right)^2+\left(M-\frac{C\,r^3}{d^3}+\frac{3\,C\,r^3}{d^3}\cos^2\phi\right)^2\right\}}$$

Or dividing all the terms by $\frac{3C\,r^3}{d^3}\cos\phi$, we have

$$\frac{3\,C\,r^3}{d^3}\cos\phi\sqrt{\left\{\sin^2\phi+\left(\frac{M\,d^3-C\,r^3}{3\,C\,r^3}\sec\phi+\cos\phi\right)^2\right\}}$$

And making $\frac{M\,d^3-C\,r^3}{3\,C\,r^3}=A$

this reduces to

$$\frac{3\,C\,r^3}{d^3}\cos\phi\sqrt{\left\{\sin^2\phi+(A\sec\phi+\cos\phi)^2\right\}}=$$

$$\frac{3\,C\,r^3}{d^3}\cos\phi\sqrt{(1+2\,A+A^2\sec^2\phi)}=$$

$$\frac{3\,C\,r^3\,A}{d^3}\sqrt{\left\{\frac{1+2\,A}{A^2}\cos^2\phi+1\right\}}$$

Or re-establishing the value of A, and calling the natural magnetic intensity $M=1$, in which case $C=1{\cdot}0535$, (art. 165), we have for our resultant

$$R=\sqrt{\left\{\frac{2\,d^3+C\,r^3}{d^6}\cdot 3\,C\,r^3\cos^2\phi+\frac{(d^3-C\,r^3)^2}{d^6}\right\}}\quad(17)$$

from which R may be computed.

184. For example, if $d=20$, $r=6{\cdot}4$, and $C=1{\cdot}0535$, we find

$$R=\sqrt{({\cdot}16743\cos^2\phi+{\cdot}93213)}$$

Or if all the rest remain the same, except r, and we make this $=8{\cdot}85$, the radius of an 18 inch shell, we shall then have

$$R=\sqrt{({\cdot}57288\cos^2\phi+{\cdot}82582)},\text{ or}$$
$$R=\sqrt{({\cdot}57288\sin^2\lambda+{\cdot}82582)}$$

185. In order to compare these results with experiment, we must consider the vibrations of a

magnetized needle under the same point of view as those of a simple pendulum; that is, we must estimate the force as proportional to the square of the number of vibrations made in a given time; or the number of vibrations as proportional to the square root of the force; we have therefore only to give the proper values to our angle ϕ, or λ, and compute the value of R. Then, since the natural magnetic intensity has been called unity, we shall have

$$1 : \sqrt{R} :: N : n$$

where N is the number of vibrations made by a dipping needle in any given time, when removed from every extraneous magnetic action, and n the number made in the same time, when the instrument is placed in any given latitude on either of our iron balls, and at 20 inches distant from their centre.

If the distance was different, the numbers in the preceding numerical formulæ would, of course, be different also. I merely select this distance as the most convenient, and for the purpose of comparing the formula with the results obtained in our experiments (art. 174), in the last column of which table, is given the number of seconds employed by the needle (7 inches in length) in making 40 vibrations.

The time was taken by an instrument which gave the intervals to 40ths of seconds; but I have not

attempted to register nearer than to quarter seconds. (See art. 174).

186. *Table showing the computed and observed intensities of a Dipping Needle in different positions, with respect to an* 18 *inch Shell.*

Value of R^2 or of $\cdot 57288 \sin^2 \lambda + \cdot 82582$			Time of 40 vibrations detached from Iron.	Computed time of 40 vibrations in different Latitudes.	Observed time in different Latitudes.
Latitude.		Value of R^2			
°	′		″	″	
0	0	·82582	120·	125·78	125·3
15	0	·86419	Ditto	124·46	124·44
30	0	·96940	Ditto	120·95	121·3
33	30*	1·0000	Ditto	120·00	120·3
45	0	1·11226	Ditto	116·85	116·5
60	0	1·25548	Ditto	113·36	113·58
75	0	1·36033	Ditto	111·12	112·3
90	0	1·39870	Ditto	110·35	110·25

* The proper value of λ when $R = 1$, that is, when $\cdot 57288\ \lambda^2 = \cdot 17418$ is $\lambda = 33° 28'$.

187. The remarkable approximation to equality in the computed and observed times of vibration, in the last two columns of the above table, is one of the best tests we have yet met with for comparing the hypothesis upon which our investigation is founded, with the interpretation usually given to the theory of Coulomb. According to the latter, every ball or mass of iron becomes, by induction from the earth, a temporary magnet, having its north and south poles at its extremities, or at those

of the line corresponding with the dip of the needle in the place of observation. Whereas, agreeably to the hypothesis I have advanced, the latent magnetism of the ball has merely polarity given to it; but it is distributed over the surface, and has its centres of action indefinitely near to each other in the centre of the ball.

188. On discussing this question with a philosopher of eminence, he maintained an opinion that the ball which I had employed acted from two poles, at the extremities of that diameter which corresponds with the dip of the needle, and suggested an experiment that should demonstrate the fact.

This was, to take a vessel of water, and to file a piece of soft iron with a new file, so that the dust of the iron should be distributed on the surface of the water; I was then to bring the north pole of the ball nearly in contact with the surface of the water, and the motion of the filings was to indicate the existence of the pole in question.

I performed this experiment in two or three different ways, but I could never distinguish the least motion in the filings. This, however, after all would be considered only a negative demonstration of what I wished to establish; but the preceding results are, I conceive, fully conclusive and satisfactory.

189. For the better illustration of the conclusion

I am desirous of drawing from the above tabulated experiments; I have given, in (fig. 16) a delineation to scale of the position of the needle *n s*, in respect to the ball N S in each of the situations noted in the preceding table; by which it will be seen how closely the south end, *s*, of the needle approached (in its position lat. 90°) what has been called the north pole of the ball; consequently, the acceleration of the needle ought, in this position, to have been much greater than we have found it; had the action taken place between N and *s*, or if N were a condensed centre of action, such as the hypothesis in question supposes. Whereas by referring the whole to a compound central action, we find the most accurate agreement between the observed and computed intensities.

190. I am the more anxious to establish this point, in consequence of its immediate connection with the method I have proposed for correcting the errors of a ship's compass, which has been objected to, on the ground that according to the theory we have been controverting, the central action of all the iron on board would not remain constant under all dips and in all parts of the world; but if the hypothesis I have advanced be correct, then the central action of any irregular mass of iron will be in the centre of attraction of its surface, whatever may be the magnetic direction,

and must necessarily remain the same, while the iron and the point from which its action is estimated preserve the same relative situation; as is the case with the iron of a vessel and its compass, at least, with the exception of those small changes of position which may, for the sake of convenience, take place in the course of the voyage; but these will never materially affect the position of the general centre of the whole mass.

SECTION III.

ON THE MAGNETIC ACTION OF BARS OF IRON.

191. It will be observed that we have hitherto confined our investigation to the laws of action of spherical bodies, which possess the singular quality of having their centre of attraction in the centre of the mass; consequently the former remains fixed in position, at whatever distance the compass may be placed from the ball. This, however, is not the case with any other body; and I was therefore desirous of ascertaining, by experiment, how nearly the deduction I had already made from spherical bodies agreed with those of a different form.

192. These deductions, as we have already seen, amounted to this,—that the magnetic attraction

of any iron body, may be referred to the action of two centres indefinitely near to each other in the general centre of attraction of the surface of the body: viz. that point into which, if all the matter of the surface were collected, its action on a given point (the centre of the compass) would be the same as the action of the whole body in its natural form.

In order, therefore, to compute the action of a bar of iron (considered as a line) on the compass, we must first determine the position and distance of the centre of attraction, or at least the amount of that attraction on a given point.

This part of the computation may be effected as follows:

193. Let A B, (fig. 17) denote the given bar, C the place of the compass, or the point to which the attraction is referred. Draw the perpendicular D C and join A C, B C.

Make A D $= b$, D C $= a$, A C $= c$, and m D $= x$, any variable distance. Then, the attraction being inversely as the square of the distance, the attraction between m and c will vary as

$$\frac{1}{\overline{\mathrm{C}m}^2} = \frac{1}{a^2 + x^2}$$

Resolve this into the two directions D C, D m, and we shall have

$$\frac{a}{(a^2 + x^2)^{\frac{3}{2}}} = \text{attraction in the line C D}$$

$$\frac{x}{(a^2 + x^2)^{\frac{1}{2}}} = \text{attraction in the line D } m$$

consequently

$$\int \frac{a\,d\,x}{(a^2 + x^2)^{\frac{3}{2}}} = \frac{x}{a\,(a^2+x^2)^{\frac{1}{2}}} = \text{sum of attraction in D C}$$

$$\int \frac{x\,d\,x}{(a^2 + x^2)^{\frac{3}{2}}} = \frac{-1}{(a^2 + x^2)^{\frac{1}{2}}} + \frac{1}{a} = \text{sum of attrac. in D } m$$

$\left(\frac{1}{a}\right.$ being the correction.)

These results, when x become equal to b, that is, the whole attraction of the line D A, will be

$$\frac{b}{a\,\sqrt{(a^2 + x^2)}} = \frac{b}{a\,c} = \text{force in D C}$$

$$\frac{1}{a} - \frac{1}{c} = \text{force in D A}$$

In the same manner, denoting the lines on the other side of D, by the corresponding letters a, b', c', we shall have for the attraction of the part D B

$$\frac{b'}{a\,c'} = \text{force in D C}$$

$$\frac{1}{a} - \frac{1}{c'} = \text{force in D B}$$

The two rectangular forces, therefore, due to the whole line will be

$$\frac{b}{a\,c} + \frac{b'}{a\,c'} = \text{force in D C}$$

$$\frac{1}{c'} - \frac{1}{c} = \text{force in D A}$$

Let the former of these be denoted by m and the latter by n, then the resultant R, by the known principle of forces, will be

$$R = \sqrt{(m^2 + n^2)}$$

If now we denote by ϕ the angle which this resultant makes with the line A B, we shall have

$$\tan \phi = \frac{n}{m}$$

whence the attraction of the line A B, upon the point C, becomes determined both in quantity and direction.

194. But to apply this result to the case of magnetic attractions, we must refer to our approximate law (art. 164), from which it appears, that the tangent of deviation of a needle varies as the cube of the distance inversely (or as the $\frac{3}{2}$ power of the force directly) multiplied by the product of the sine and cosine of the latitude (or of the co-latitude) and the cosine of the longitude.

In the present case, if we suppose the bar to be placed in the direction of the dipping needle, the angle ϕ will be the co-latitude, and we have, obviously,

$$\sin \phi = \frac{m}{\sqrt{(m^2 + n^2)}}$$

$$\cos \phi = \frac{n}{\sqrt{(m^2 + n^2)}}$$

the tangents of deviation ought to vary, therefore, as

$$R^{\frac{3}{2}} \sin \phi \cos \phi \cos l$$

(l being the longitude); or substituting the above values of R, $\cos \phi$ and $\sin \phi$, this becomes

$$\frac{m\, n \cos l}{(m^2 + n^2)^{\frac{1}{4}}}$$

that is

$$\tan \Delta = A \frac{m\, n \cos l}{(m^2 + n^2)^{\frac{1}{4}}}$$

where Δ denotes the angle of deviation, and A a constant co-efficient.

195. In the experiments with which it is proposed to compare this formula, the compass was placed due east and west of the bar, consequently the longitude of its position was zero, and therefore $\cos l = 1$. Hence in this particular case, the formula is farther reduced to

$$\tan \Delta = A \frac{m\, n}{(m^2 + n^2)^{\frac{1}{4}}}$$

or

$$\tan \Delta \cdot \frac{(m^2 + n^2)^{\frac{1}{4}}}{m\, n} = A \text{ a constant quantity,}$$

whatever may be the distance of the compass, or its position, provided the longitude be zero.

196. The experiments were performed by Mr. Bonnycastle, on a bar 24 inches in length, and 1¼ inch square; inclined in the direction of the dipping needle, and the compass was placed to the east and west of the bar; first opposite to its centre, and then at every 3 inches from the centre towards the extremities, at the distances of 12 and 16 inches from the axis of the bar.

The following are the observed results, and the computed value of the constant co-efficient.

197. *Table of observed deviations and computed value of the constant quantity* A.

Distance of Compass from bar.	Distance below centre.	Observed deviations.		Value of m	Value of n	Value of $\frac{m\,n}{(m^2+n^2)^{\frac{1}{4}}}$	Value of A
Inches.	Inches.	°	′				
16	3	2	20	·07338	·00887	·002396	17·03
Ditto	6	4	25	·06860	·01699	·004384	17·62
Ditto	9	5	45	·06123	·02355	·005631	17·88
Ditto	12	6	0	·05201	·02783	·005960	17·63
12	3	5	20	·11500	·01400	·004840	19·28
Ditto	6	10	0	·10610	·02830	·008989	19·62
Ditto	9	12	0	·09251	·03949	·011521	18·45
Ditto	12	11	30	·07450	·04606	·011600	17·54
						Mean	18·13

In examining the computed value of A in the last column of the above table, it should be remembered that we have only employed our approximate formula, which, at small distances, gives rise to certain inequalities sufficient to account for the discrepance in the co-efficients in the fifth and sixth experiment; but it may be proper to observe, that the excess of these, above the mean, amounts to but a small fraction of a degree.

198. Upon the whole, therefore, I conceive that we may be allowed to cite these results as a further proof of the accuracy of the principles upon which our hypothesis is founded, and of the deduction we have made from it; viz. that the action of plain unmagnetized iron on a compass may be referred

to two poles indefinitely near to each other in the common centre of attraction of the surface of the body, and consequently as a proof of the accuracy of the method proposed for correcting the local attraction of a vessel in all parts of the world.

Supplementary Experiments to the preceding.

199. Although the results already obtained are, in my opinion, quite decisive of the question respecting the nature of the magnetic developement in iron bodies, yet, as it is difficult to conceive how it happens that the penetration takes place to equal depths in every part of the surface, I was still desirous of instituting another course of experiments that should leave no possible doubt. This was by means of a thick iron plate, which, by being made to revolve on its lower edge, would present various degrees of thicknesses to the line of the dip; and hence by observing the deviation of a compass with the plate at different degrees of inclination, we should be able to discover whether the law which hitherto has been found to obtain, held good in this case also. Mr. Bonnycastle obligingly undertook this course of experiments, and the requisite investigation, and the results will be best given in his own words.

200. "On receiving the plate* you sent me, I commenced the experiments given below, and their results I think you will find to be as satisfactory as can be wished.

"I placed the plate with its edge A B (fig. 18) parallel to the horizon, and at right angles to the meridian *a b*; and having caused it to revolve about A B until the angle D A *b* was 20°, I then observed its attraction upon a compass at O, which is in the line B A produced; this attraction was zero, as it ought to be very nearly, which renders it probable that there is but little permanent magnetism in the plate. Having ascertained this point, I next revolved the plate about A B and through the various angles D′ A *b*, D″ A *b*, &c. taking at each the deviation of the compass at O; the results of these experiments are recorded in the table which is given below; and it only remained for me to calculate theoretically the corresponding deviations in order to decide upon the point in question.

"The attraction to the plane A B E D in the directions O A, A D, is the same as that of the plane A B E′ D′ in the directions O A, A D′; or of A B D″ E″ in the directions A O, A D″, &c.

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* The plate was 12 inches square and half an inch thick, of malleable iron.

and, therefore, the deviation of the needle will vary as the same function of the angle D A *b*, whatever be the form of the plate in question, or the quantity of its attraction, provided that the depth to which the magnetic action penetrates is always the same; but if this depth depends on the position of the surface with respect to the dip, it is manifest that in the various positions of the plate which are represented above, the law of attraction will vary considerably from that obtained on the hypothesis of a constant penetration.

"That this hypothesis is correct, will, however, readily appear from the following calculation of the deviations according to theory, and the comparison of the results with those actually observed.

"Let C (fig. 19) be the centre of the compass; C *o* the line about which the plate revolves; E the centre of attraction; S N the dip; A D B F the equator; and *a b*, the meridian, join C E; draw E *d* perpendicular to A B, and join C *d*. Then the supplement of the co-latitude is, evidently, equal to C E *d*; and the longitude is equal to *d* C *o*: but by what has been observed above, the centre of attraction E, is always found in the circle B E A; and, therefore, as C *o* is invariable, C E will be also invariable; and (putting co-lat. $= \phi$) $\cos \phi$ will vary as E D; that is as the cosine of S E. For the same reason $\sin \phi$ will vary as C *d*; and $\sin \phi \cos l$ will vary as C *d* $\cos$ *d* C *o* = C *o*; which is

constant, and consequently sin ϕ cos ϕ is also constant.

"But the approximate formula for the deviation is

$$\tan \delta = \text{A}. \frac{\cos l \sin \phi \cos \phi}{d^3}$$

and in this case d^3, and the rectangle cos l sin ϕ are constant, and cos ϕ varies as cos S E, or as cos ϵ (putting S E = ϵ); wherefore

$$\frac{\tan \delta}{\cos \epsilon} = \text{M}.$$

where M is a constant quantity.

Angle ϵ	Observed deflection.	Value of M.
110	5° 0′ W	·2558
90	0 0	....
50	9 25 E	·25807
20	14 35	·27687
0	15 45	·282029
−30	12 20	·28542
−70	5 40	·29012

"*Note.* The distance of the compass from the plate was six inches, and the deflections were the means of two sets of observations made with opposite edges of the plate, the results of which how-

ever differed in no case more than 20 minutes from each other.

"As the first and last values of M ought, evidently, to be the same, it appears that the difference between the numbers in the last column arises from a permanent magnetism in the plate."

201. When it is remembered that the above numbers are obtained from an approximate formula, and that the aberrations are such as would be compensated by a change of only a few minutes in the observed angles, there can, I think, be no doubt that the developement of magnetism takes place according to the hypothesis I have advanced; viz. to only a certain depth, and to the same depth, whatever be the position of the body or the thickness of the metal, provided it exceed a certain quantity.

SECTION IV.

APPLICATION OF THE PRECEDING FORMULÆ TO THE MAGNETISM OF THE TERRESTRIAL SPHERE.

202. In the foregoing investigations the needle has been supposed to be under the influence both of the earth and of the iron sphere: let us now

examine what those results will become by supposing the terrestrial action which we have denoted by M (art. 160) to cease. That is, we shall conceive the ball to retain its magnetic power, but the force M to become zero. In this case the ball will, *in miniature*, resemble the action of the terrestrial globe, and the laws which we thus deduce ought to be analogous to those obtained from observations in different parts of the earth; or at least if we find the laws thus obtained similar to the former, we shall have strong reasons to conclude that the action of the ball and that of the earth are perfectly analogous to each other; and since we have determined experimentally, that in the former case this action proceeds from the magnetic fluids covering the surface of the ball, so we ought in the latter to draw the same inference; that is, we ought to consider the terrestrial magnetism as due only to the surface, although the centres of action of the two fluids may be referred to two points indefinitely near to each other, and to the geometrical centre of the earth itself.

203. With a view to the investigation here alluded to, we must recur to our formula (7), which expresses the deviation of a needle freely suspended from its natural direction N′ S′, or N S, (fig. 13) the magnetic axis of the sphere. We have therefore only to consider $M = o$, and this becomes immediately

$$\tan \Delta = \frac{3 \cos \phi \sin \phi}{3 \cos^2 \phi - 1}$$

Let this angle be represented by N Pn, (fig. 13) which is equal to N o P the co-latitude, $(= \phi)$ + o Pn (= complement of the dip); denoting therefore the complement of the dip by δ' we shall have

$$\tan (\phi + \delta') = \frac{3 \cos \phi \sin \phi}{3 \cos^2 \phi - 1} = \frac{3 \cos \phi \sin \phi}{2 \cos^2 \phi - \sin^2 \phi}$$

or,

$$\frac{1 - \tan \phi \tan \delta'}{\tan \phi + \tan \delta'} = \frac{2}{3} \cot \phi - \frac{1}{3} \tan \phi$$

This by reduction becomes

$$2 \tan \delta' (\cot \phi + \tan \phi) = 1 + \tan^2 \phi$$

or,

$$2 \tan \delta' = \tan \phi$$

or representing the dip by δ,

$$2 \cot \delta = \cot \lambda$$

whence

$$\tan \delta = 2 \tan \lambda$$

That is, the tangent of the dip is equal to double the tangent of the magnetic latitude. A law which has been obtained, by a comparison of magnetical observations in different parts of the earth with each other.

204. We may arrive at this conclusion in a more simple way, by referring to our two forces (3) and (4), only rejecting M, as above, out of the latter, which thus become

$$A^3 \cos \phi \text{ —— in the direction } Po,$$
$$\left\{ \begin{array}{l} -A \text{ —— in the direction S N} \\ \text{or } A \text{ —— in the direction N S} \end{array} \right.$$

taking A to denote the co-efficient $\frac{C\,r^3}{d^3}$

Draw P H (fig. 13) perpendicular to Po which will thus represent the horizon, or a line parallel to it; consequently S′ P H will be the latitude $= \lambda$, and H Pn = the dip $= \delta$.

Resolve the latter force S N into two, one in the direction P H $= A \cos \lambda$, and the other into oP $= A \sin \lambda$; by this means our rectangular forces will be,

$$A \cos \lambda \text{ —— in the direction P H}$$
$$A\,(3 \cos \phi - \sin \lambda) \text{ in the direction P } o$$

But $\cos \phi = \sin \lambda$, they therefore reduce to

$$A \cos \lambda \quad \text{in the direction P H} \ldots\ldots (18)$$
$$A \,.\, 2 \sin \lambda \text{ in the direction P } o \ldots\ldots (19)$$

and consequently the angle of their resultant, or the angle it forms with the horizon, will be

$$\tan \delta = \frac{A\ 2 \sin \lambda}{A \cos \lambda} = 2 \tan \lambda \ldots\ldots\ldots (20)$$

the same result as above.

205. With respect to the magnetic intensity in any place, this may be deduced from our formula (17) by making $M = o$, but it is more readily drawn from the above forces (18) and (19).

For here we have immediately, for the resultant of those forces, or for the intensity in the natural direction (which we may denote by I).

$$I = A \sqrt{(4 \sin^2 \lambda + \cos^2 \lambda)} = A \sqrt{(3 \sin^2 \lambda + 1)} \quad .. (21)$$

while that for the horizontal needle I′, is expressed in the first force itself, viz.

$$I' = A \cos \lambda \quad ..... (22)$$

The intensity therefore of a dipping needle, in any part of the earth, varies as the square root of three times the square of the sine of the magnetic latitude *plus* unity; and that of the horizontal needle varies as the cosine of the magnetic latitude.

206. In order to express these values in terms of the dip, we have only to employ the relation already established between the latitude and dip, namely,

$$\tan \delta = 2 \tan \lambda$$

or

$$\frac{\sin^2 \delta}{\cos^2 \delta} = \frac{\sin^2 \delta}{1 - \sin^2 \delta} = \frac{4 \sin^2 \lambda}{1 - \sin^2 \lambda}$$

whence

$$\sin^2 \lambda = \frac{\sin^2 \delta}{4 - 3 \sin^2 \delta}$$

In the same manner

$$\cos^2 \lambda = \frac{4}{3 + \sec^2 \delta}$$

and substituting these values in the preceding expressions for I and I′, we have

$$I = 2 A \sqrt{\frac{1}{4 - 3 \sin^2 \delta}} \quad ..... (23)$$

$$I' = 2 A \sqrt{\frac{1}{3 + \sec^2 \delta}} \quad ..... (24)$$

That is, the natural magnetic intensity varies inversely as the square root of 4 *minus* three times

the square of the sine of the dip; and that of the horizontal needle, inversely as the square root of 3 increased by the square of the secant of the dip, which are both known laws of terrestrial magnetism.

207. These formulæ have been compared by Captain Sabine with the results of observations made during the interesting voyage of Captain Parry through Barrow's Strait, of which the following is an abstract. See Appendix to Parry's voyage.

208. London, dip 70° 33′; the mean time of making 100 vibrations with the dipping needle was 8^m 02″ or 482″. Ditto, on the return of the Expedition, 480″. Winter harbour, dip 88° 43′; the mean time of making 100 vibrations was 7^m 26″·25 = 446″·25. We ought, therefore, to have by formula (24)

$$(4 - 3 \sin^2 70° 33')^{\frac{1}{2}} : (4 - 3 \sin^2 88° 43')^{\frac{1}{2}} :: 481^2 : 446{\cdot}25^2$$

And we have in actual numbers, for the last term, $\overline{447{\cdot}84}^2$, which is as near an approximation as we could reasonably expect.

209. Again the number of seconds in making ten vibrations with two horizontal needles, were as follow:

			″
Needles	No. 2.	Sheerness, dip 69° 55′, vib. 10, time	90
	No. 3.	Ditto	85
Needles	No. 2.	Ice Davis' Strait, dip 83° 04′	151·5
	No. 3.	Ditto	143·75

Needles	No. 2.	Baffin's Bay, dip 84° 30′	163·0
	No. 3.	Ditto	152·5
Needles	No. 2.	Winter Harbour, dip 88° 43·5′....	329·0
	No. 3.	Ditto	318·0

210. Comparing the results at the greatest of the above dips with the least, we ought to have, taking the means of the two needles,

$$(3 + \sec^2 69° 55')^{\frac{1}{2}} : (3 + \sec^2 88° 43·5')^{\frac{1}{2}} : \overline{87·5}^2 : \overline{323·5}^2$$

and we have in actual numbers for our fourth term 317·76, which is a very close approximation.

In like manner we should have

$$(3 + \sec^2 69° 55')^{\frac{1}{2}} : (3 + \sec^2 84° 30')^{\frac{1}{2}} : \overline{87·5}^2 : \overline{157·7}^2$$

and the last term is $\overline{154·59}^2$

also

$$(3 + \sec^2 69° 55')^{\frac{1}{2}} : (3 + \sec^2 83° 04')^{\frac{1}{2}} : \overline{87·5}^2 : \overline{147·6}^2$$

but the computation gives the last term $\overline{138·29}^2$

This is a greater aberration than in the preceding proportions, but still, a very slight error in taking the dip would be amply sufficient to account for it.

211. This relation between the dip and intensity of vibration has suggested to Captain Sabine the idea of obtaining the former by means of the latter.

For example, let D denote the dip in any place where it is known, and let n be the number of vibrations made by a finely suspended horizontal needle in the same place, in any given time. Let also N be the number of vibrations which the same

needle makes in the same time, in any other place where the dip (d) is required; then

$$(3 + \text{see}^2\, \text{D})^{\frac{1}{2}} : (3 + \text{see}^2\, d)^{\frac{1}{2}} :: \text{N}^2 : n^2$$

or

$$(3 + \text{see}^2\, d)^{\frac{1}{2}} = \frac{(3 + \text{see}^2\, \text{D})^{\frac{1}{2}}\, n^2}{\text{N}^2}$$

or

$$3 + \text{see}^2\, d = \frac{n^4}{\text{N}^4}\,(3 + \text{see}^2\, \text{D})$$

whence

$$\text{see}\, d = \left\{ \frac{n^4}{\text{N}^4}\,(3 + \text{see}^2\, \text{D}) - 3 \right\}^{\frac{1}{2}} \ldots\ldots \quad (25)$$

where n, N, and D, being given, d also becomes known.

212. It is to be observed that n and D, when once determined for any given place, as for example London, will be constant; let therefore $n^4\,(3 + \text{sec}^2\, \text{D}) = \text{A}$, then the above becomes simply

$$\text{see}\, d = \sqrt{\frac{\text{A}}{\text{N}^4} - 3} \quad \ldots\ldots\ldots \quad (26)$$

Or, if instead of counting the number of vibrations made in a given time, we take the time that the needle is in performing a given number of vibrations, (which is the best where we have a good stop-watch, or other instrument for marking shorter intervals of time); then denoting the times by T and t, the above formula is transformed into

$$\text{see } d = \left\{ \frac{T^4}{t^4} (3 + \text{see}^2 D) - 3 \right\}^{\frac{1}{2}} \ldots\ldots (27)$$

which, like formula (25) is reducible to the simple form

$$\text{see } d = \sqrt{T^4 A' - 3} \ldots\ldots\ldots\ldots (28)$$

after the dip D, and time t, have been ascertained in the first place of observation.

213. We may take as examples the numbers given in (art. 209), where it appears that with the dip, 69° 55′, the mean time of making ten oscillations was 87·5″; whence

$$A = \frac{3 + \text{see}^2\ 69° 55'}{\overline{87 \cdot 5}^4} = \cdot 00000019584$$

Now at Winter Harbour, Melville Island, the mean time of making 10 vibrations was 323·5″, whence we have for the dip at the latter place

$$\text{see } d = \sqrt{(\overline{323 \cdot 5}^4 + \cdot 00000019584 - 3)} = 88° 46'$$

In Baffin's Bay,

$$\text{see } d = \sqrt{(\overline{157 \cdot 7}^4 + \cdot 00000019584 - 3)} = 84° 43'$$

In Davis' Strait,

$$\text{see } d = \sqrt{(\overline{147 \cdot 6}^4 + \cdot 00000019584 - 3)} = 83° 57'$$

Now the actual observed dips in those places were 88° 43′, 84° 30′ and 83° 04′; consequently the approximations by computations, at least with the exception of the last, are as close as we have any reason to expect, in calculations of this kind.

214. Having shown these formulæ to Captain Owen, the scientific commander of the Leven, he

requested Mr. Jones, of Charing-cross, to supply him with an instrument proper for the purpose; it was one of Captain Kater's diamond formed needles, suspended in a common binnacle bowl hung in gymbols.

The needle was suspended by a fine fibre of untwisted silk, from an arc rising above the bowl, but it was kept to its centre by means of a fine point and agate cap on which it slightly rested; the silk suspension having an adjustment by which this could be effected.

We made trial of the instrument at Northfleet, at a time when, in consequence of the wind being very high, there was considerable motion in the vessel, and from which alone the needle was in a continual state of oscillation in short arcs of about 5° on each side the north.

By means of a watch, which registered time to 12ths of seconds, we counted the time of the needle making 100 vibrations, and found it, by a mean of several trials, to be $6^m\ 22{\cdot}4''$. We have therefore

$$A' = \frac{\text{see}^2\, D + 3}{(382''{\cdot}4)^4} = {\cdot}00000000056$$

and consequently in any other case the dip will be found by the formula

$$\text{see}\, d = \sqrt{({\cdot}00000000056\, T^4 - 3)}$$

T being the number of seconds in which the needle will perform 100 vibrations.

215. On the magnetic equator sec $d = 1$, and we find $T = 290''\cdot7$; there ought to be therefore $91''\cdot7$ difference in the time of performing 100 vibrations between Northfleet and the magnetic equator, or at a mean there ought to be about $1''\frac{1}{3}$ difference in the time of making the 100 vibrations for every change of one degree in the inclination of the needle, which, if the principle be correct, is quite sufficient for determining the dip on shore to a certain degree of accuracy.

When I first mentioned these formulæ to Captain Owen I was in hopes the method might be employed on ship board: but on farther consideration, it was obvious that notwithstanding we might always make the experiments when the ship was in the meridian, yet the intensity of the directive power of the iron would vary with the magnetic latitude, and consequently with the dip.

When the vessel is in the meridian it is under the influence of one force only, which is expressed by the denominator of formula 8, (art. 161); where it is obvious that not only M, the natural magnetic intensity, will vary with the dip, but that the angle ϕ, and consequently the entire expression will vary in like manner. The method, therefore, is only applicable to experiments on shore, although the approximation may be very close on board.

216. I shall only add farther on this subject, that I do not consider the silk suspension actually

necessary in an instrument of this kind; and I should prefer a simple light bar needle to the diamond formed needle described above. The great object is to have as many vibrations as possible in a given time, and I think this is more likely to be obtained with the common needle than with the more powerful one of Captain Kater's form.

Note. To preserve the needle in a constant state of magnetic saturation it should be kept, when not in use, connecting the extremities of a magnet in the form shown in figure 20.

The needle ought also to have a sliding slip of brass on its northern or southern branch in order to adjust it to horizontality.

SECTION V.

ON THE SITUATION OF THE TERRESTRIAL MAGNETIC AXIS, AND ON ITS ANNUAL MOTION.

217. HITHERTO we have found a very close approximation between the laws of magnetism appertaining to a simple iron ball and the observed magnetic phenomena of the earth. If the same analogy had place in all cases, there can be no doubt, that we ought to be able to compute the actual direction of the terrestrial magnetic axis. For since we have seen that the tangent of the dip

is equal to double the tangent of the magnetic latitude; this latter, and consequently the magnetic co-latitude of any place on the earth, would be given supposing the dip known.

Having thus the magnetic polar distance, and the variation indicating the direction of the pole itself, the actual situation of the latter may always be computed when the dip and variation are known.

218. To render this more obvious, let N S fig. 21. represent the terrestrial poles, and π the magnetic pole, and let L be any place on the earth where the dip and variation are given. Then by means of the dip we determine the magnetic co-latitude π L, the terrestrial co-latitude N L is also supposed known, as well as the variation or angle N L π. Hence in the spherical triangle N L π, we have two sides and the contained angle, to find the side N π, (the terrestrial co-latitude of the magnetic pole) and the angle π N L, (the longitude of the same) as referred to the meridian N L.

Consequently if the earth had a decided axis of polarization, such as appertains to the iron ball, we ought to be able to find its situation by the computation above indicated, and it ought to be the same, at the same time, in whatever part of the earth the dip and variation are observed.

219. In order to examine the question under this point of view, I have selected a certain number of observations on the dip and variation of the needle, on which the greatest reliance may, I conceive, be placed, and have computed the situation of the magnetic pole agreeably to the above principles; but it will be found that there is here no longer that coincidence between the results and observed phenomena which has attended our preceding comparisons.

220. *Computed situation of the magnetic north pole of the earth, from observations on the dip and variation of the needle in different places.*

Place of Observation.	Date.	Terrestrial latitude and longitude.		Dip.	Variation.	Computed place of magnetic pole.		Names of Observers.
		Latitude.	Longitude.			Latitude North.	Longitude West.	
		° ′	° ′	° ′	° ′	° ′	° ′	
Tristan da Acunha	1821	37 0 S	12 10 W	37 53 S	12 00 W	70 56	49 83	Captain Marryat
Trinidad	Ditto	20 30 S	29 00 W	10 27 N	5 00 W	73 59	47 20	Ditto
St. Jago	1820	14 54 N	23 32 W	48 00 N	15 55 W	69 37	67 4	Mean of Capt. Marryat and Lieut. Mudge's observations*
Teneriffe	Ditto	28 28 N	16 16 W	58 22 N	20 47 W	69 49	69 14	
Maderia	Ditto	32 38 N	17 54 W	63 47 N	23 7 W	68 4	65 26	
Madrid.....	1799	40 25 N	3 29 W	67 41 N	19 59 W	72 47	50 33	Humboldt
Paris	1814	48 50	2 20 E	68 36 N	22 34 W	75 31	67 4	Bouvard
London	1818	51 31 N	0 0	70 34 N	24 30 W	75 2	67 41	Kater the dip
Berlin	1805	52 32 N	13 21 E	69 53 N	18 2 W	79 2	70 44	Humboldt
Copenhagen	1813	55 41 N	12 35 E	71 26 N	18 22 W	79 43	67 38	Wleugel
Davis' Strait	1820	64 00 N	61 50 W	83 43 N	60 20 W	67 37	94 26	Parry
Regent's Inlet....	Ditto	72 45 N	89 41 W	88 26 N	118 16 W	71 10	98 16	Ditto
Baffin's Bay, on ice	Ditto	73 00 N	61 30 W	84 30 N	82 2 W	71 13	97 3	Ditto
Possession Bay ..	Ditto	73 31 N	77 22 W	86 4 N	108 46 W	69 40	99 10	Ditto
Melville Island ..	Ditto	74 47	110 48 W	88 43 N	127 46 E	73 12	102 46	Ditto

* Captain Marryat, of H. M. S. Rosario, having obligingly furnished me with his observations on the dip at the above places, and Lieut. Mudge having done the same, I have drawn the above numbers from the mean of both observations, which agreed very nearly with each other.

221. Although in determinations of the dip and variation of the needle we cannot expect the utmost accuracy, yet it is very obvious, from the preceding results, that the aberrations in the latitude and longitude of the magnetic pole are much greater than can be attributed to errors in observation. It will be seen that the place assigned to it differs in longitude as much as 55° between one set of observations and another, and as much as 10° in latitude. It will also be observed that the more we approach the north, and west, the more westerly we find the place of the pole; and the more easterly the place of observation, the greater is the latitude of the pole. In short it is evident from the few examples we have taken, that every place has its particular polarizing axis, which probably in all cases fall within the arctic circle, and that this is the narrowest limits we are able to assign.

222. Instead, therefore, of the magnetism of the earth possessing that degree of uniformity which appertains to a perfectly formed iron ball, it may be rather said to resemble that species of action which we might expect to find in an irregularly formed mass of iron, approximating in its general character to that of a globe, but not perfectly such; and if the magnetism of the earth be due to the distribution of iron in its interior, we ought in fact rather to expect *a priori* such a kind of action than that which belongs to a perfectly formed iron sphere.

It is true that the observations we have used were not made simultaneously, and that a change is perpetually going on in the direction of the axis of polarization, which circumstance alone would give rise to some discrepances; but not, as will be seen in the following article, to the amount shown in the preceding table.

Every place, therefore, appears to have its proper poles; and the only limit we are enabled to assign to their situations is, that as far as observations have yet been carried, they appear to fall somewhere within the two frigid zones, but varying through all possible degrees of longitude and latitude within these limits.

These aberrations being however attributed to local inequalities in the distribution of the ferruginous parts of the terrestrial sphere, we ought still to expect a certain degree of uniformity in the annual changes which take place in the situations of the poles of any particular place; supposing these changes to arise from some general cause acting equally on all. Let us then examine the circumstances attending the annual variation of the needle, and ascertain how far this phenomenon is reducible to determinate laws.

Of the Annual Variation.

223. It is not my intention to offer in this place any conjecture respecting the cause of this annual

change; it is sufficient that we know the general fact, viz. that the polarizing axis of any particular place is perpetually changing its direction; this direction being known at certain times, and the rate of change supposed uniform, we shall be able to compute what it ought to be at others, and then by comparing these results with known observations, we shall be enabled to judge of the accuracy of our first assumptions.

We have seen (art. 220) that the longitude of the polarizing axis which governs the needle in London, was 67° 41′ W in 1818, and that its latitude was 75° 2′ N. Now in 1660 the variation in London was nothing; consequently we have a right to assume that in the latter year the longitude of the pole was zero: and if we farther suppose that the motion during this time has been uniform, and made at the same distance from the terrestrial pole, we shall find it amount to about 4° 14′ in 10 years: and hence it will be easy to compute the situation of the pole, and what ought to have been the dip and variation of the needle in London from the year 1660 to the present time, agreeably to these suppositions.

224. For example, let N S E Q (fig. 21) represent the terrestrial sphere, N its north pole, π the magnetic pole which governs the direction of the needle in London; S L N the meridian of the latter place, and *pp* the parallel of latitude in which the pole π

revolves. Then in the triangle π N L, the arc N L, or the co-latitude of London, is known, and π N, the co-latitude of the magnetic pole is supposed to be given, as also the angle π N L, the latter angle increasing at the rate of 4° 14′ for every 10 years, beginning from the year 1658 or 1660, when the pole π corresponded with the meridian S L N.

So that in the spherical triangle π N L, we have always the two sides π N, L N, and the included angle π N L given, to find the angle N L π, or the variation. And as, in this computation, the two sides π N, N L are constant, and only the angle π N L variable, we may give for this computation the following rule:

"To the co-tangent of half the angle π N L add the constant log. 1·65642; find the angle of which the sum is the tangent, and call it arc (A). To the same co-tangent add the log. 0·03987, and find the arc of which the sum is the tangent, and call it arc (B)."

Then B — A will be the variation, or angle π L N.

To find the dip we must compute the arc π L, and then 2 cotan. π L = tan of the dip.

By this rule the following variations have been computed.

225. *Table of computed and observed variation in London, from the year* 1660 *to* 1818.

By computation.					By observation.					
Year.	Variation.		Dip.		Variation.		Dip.		Year.	Authority.
	°	′	°	′	°	′	°	′		
1658, or 1660	0	0			0	0			1658, or 1660	Bond
1670	2	44			2	30			1672	Halley
1680	5	25					73	30	1676	Bond
1690	7	59			6	0			1692	Halley
1700	10	26								
1710	12	43								
1720	14	47	76	27	14	17	74	42	1723	Graham
1730	16	41								
1740	18	20			17	0			1745	Graham
1750	19	47			17	48			1748	Ditto
1760	21	1								
1770	22	4	73	40	21	9	72	19	1773	Heberden
1780	22	54	73	18	23	17	72	8	1786	Gilpin
1790	23	33	72	39	23	39	71	53	1790	Ditto
1800	24	1	71	58	24	3	70	35	1800	Ditto
1810	24	18	71	15	24	11			1809	Ditto
1818	24	30	70	34	24	30	70	34		Kater the dip

226. Although there is not in the above table that coincidence between the computed numbers and those derived from observation, which will enable us to come to any positive conclusion; yet the agreement is too close, particularly in the variations, for the last 50 years, during which time we may suppose the observations to have been made with greater accuracy, to allow us to suppose that it is entirely accidental. On the contrary; I think there can be no doubt that a motion very

similar to that we have supposed, actually takes place, although some of its elements may have been erroneously assumed. At the same time, however, great allowance is to be made for the errors and uncertainties of magnetical observations. Our authorities are, it is true, derived from the best sources, but this will not ensure us against local inequalities. Of this we have a remarkable instance in the last volume of the Philosophical Transactions, where the dip is stated to be at this time 71° 36′; which is unquestionably more than a degree greater than it really is in the neighbourhood of London, although it may be correct for the Royal Society Rooms; viz. some iron in the building, or some other hidden cause, may increase the inclination at least a degree beyond what the same would be found in the open fields.

227. That the dip at this time does not exceed 70½° is proved by a number of independent observations. For example, Captain Kater found the dip in the Regent's Park, in 1818, to be 70° 34′. Captain Sabine, on his return from the arctic voyage of that year, found it still the same, and still less prior and subsequent to his second voyage. I have also taken the dip at Woolwich with four different instruments, and the results of my mean observations have always fallen within the limits of 70° 35′ and 70° 17′; there cannot therefore be the least doubt that the dip published in the last

volume of the Philosophical Transactions, is in error at least a degree.* And if in the present day, when magnetism is becoming a mathematical science, we are liable to uncertainties of this kind, we may easily reconcile ourselves to the discrepances between the computed and observed variations and inclinations, which appear in the preceding table, without suspecting the hypothesis on which the computations are founded to be, in its general principle, erroneous.

228. We have indeed another remarkable coincidence highly favourable to the supposition of an uniform motion of rotation of the polarizing axis; which is, by computing the time when the needle ought to have its greatest westerly variation and commence again its return towards the true meridian, and comparing the result with observations.

It is obvious from the principles on which we have proceeded, that the variation will be the greatest when L π N (fig. 22) is a right angle. We have therefore in this case the side π N = 14° 58′, and the ∠ L π N = 90°, to find the angle π N L, which is found to be 70° 23′; the variation therefore ought to be the greatest when the longitude of the magnetic pole is 70° 23′ W.

Now we have seen that this longitude was 67° 41′ in 1818, and as the pole revolves at the rate

* See the following postscript.

of 4° 14′ in ten years, it follows that the longitude will be 70° 23′, some time in the year 1823; and consequently, in that year, the variation ought to be at its maximum; according to the principles and data on which we have proceeded; and as there is every reason to believe that the needle has already attained its maximum of westerly variation, and is about to return again towards the north,—the agreement in this case between computation and observation, is perhaps more satisfactory than the uncertain nature of our data could have led us to expect.

Postscript to the above Articles.

229. While these sheets have been in the hands of the printer, Captain Sabine has favoured me with a copy of an article printed for the next part of the Philosophical Transactions, in which the question of the dip of the needle in London is examined at considerable length. First, the dip was taken by a needle acting on a new principle suggested by Professor Meyer. Secondly, it was computed by a formula proposed by La Place, depending for its data on the time that the needle occupies in making a given number of vibrations when in the plane of the meridian, and in a plane perpendicular to it. And thirdly, by a method suggested by Captain Sabine himself, depending like the former on the times of vibration; but

with the needle in both instances in the meridian; viz. first in its natural dipping position, and secondly by suspending it from one extremity of its axis, and using it thus as a horizontal needle. And the several means deduced from all these different methods were as follow:

	°	′
With Meyer's needle.....	70	02·9
La Place's formula	70	04
Capt. Sabine's formula	70	02·6
3)	210	9·5
General mean....	70	3·2

The dip it appears is therefore nearly half a degree less than we have reckoned it to be in 1818, and more than a degree and a half less than the amount stated in the last volume of the Philosophical Transactions.

It is to be observed, however, that the dip we have used (viz. 70° 34′) was taken in the year 1818; and that according to the principles of calculation adopted in this chapter, this angle ought to diminish as the longitude of the magnetic pole increases. Let us therefore compute what the dip ought to be at the beginning of next year, (1823) at which time we have seen that the variation will be at its maximum, and the longitude of the magnetic pole 70° 23′: that is, referring to (fig. 22.) we shall have in the right angled triangle π N L, the side N π given, = 14° 58′; the angle

π N L = 70° 23′ to find L π the magnetic co-latitude of London; and consequently the magnetic latitude becomes known. And hence again the dip from the formula

$$\tan \text{dip} = 2 \tan. \text{mag. lat.}$$

Now

$$\tan \pi L = \sin \pi N \tan . \pi N L$$

or

$$\tan \pi L = \sin 14° 58' \times \tan 70° 23' = \tan 35° 56'$$

whence

$$\text{mag. lat. of London} = 54° 04' \text{ in } 1823.$$

and

$$2 \tan 54° 04 = \tan 70° 05'$$

the dip at that time.

The dip therefore as computed on our hypothesis ought to be 70° 05′, in the year 1823; and it has been found, from the most accurate observations ever yet made, to have been 70° 03′·2 in September 1821, a coincidence, or exceedingly close approximation, which could scarcely have been expected; and which will, I am persuaded, be duly estimated by the candid philosophical inquirer.

Captain Sabine, by comparing the present dip with the dip 47 years back, finds the mean annual diminution to be about 3′. According to our hypothesis the dip has not an uniform decrease, but is changing now more rapidly than it has ever before done since magnetical observations have been made. Its decrease during the last 5 years,

has been nearly half a degree; and if our principles be correct it ought to decrease nearly the same during the next 5 years; a short time therefore will either confirm or refute the hypothesis on which we have founded the preceding computations. Agreeably to which we ought to find in

1828, the variation 24° 29′ dip 69° 43′
1833, ... 24 26 .. 69 21

The dip, therefore, is at present changing more rapidly than the variation; and it will continue to decrease with the latter for about 260 years, when the longitude of the magnetic pole will be 180°; the variation will therefore then be nothing, and the dip only 56°, which will be its minimum, they will then both increase together for the next 260 years, when the needle will have its greatest easterly variation, and will then again return towards the north, the variation decreasing, but the dip still increasing, for 165 years longer; viz. till about the year 2510, when the magnetic pole will be again on the meridian of London; the variation will be zero, and the dip being then at its maximum will amount to 77° 43′.

Such at least are the results arising out of the hypothesis on which the preceding calculations are founded. They are unquestionably in some measure speculative, and are only given as such; but I may perhaps be permitted to say that as far as comparison could be made with well authenticated

observation it has been done, and the approximations towards coincidency have been throughout more favourable than, from the nature of the inquiry, we could have had any reason to expect. Further comparisons may also still be made, and a few years will be sufficient to confirm or refute the hypothesis, of an uniform motion of rotation in the terrestrial polarizing axis.

It is however proper to state, that if we had carried our computations back to the 16th century, and compared them with observation, the agreement would not have been found so close as in the cases mentioned in the preceding table; but I think it may be questioned how far these observations may be depended upon; one of which out of the only three we have recorded, being made previous to the time of the variation in the variation being known. At the same time I am by no means disposed to assert, that the elements of the motion we have assumed are perfect. It is to be observed that we have deduced them from two observations only; viz. the dip as taken by Captain Kater in 1818, and the variation at that time; it is from these only we have determined the latitude and longitude of the magnetic pole, and thence by assuming the longitude to have been zero in 1660, we have determined the annual motion. I have little doubt that we should have found a nearer approximation by taking the polar distance of the

magnetic pole greater or less than we have done; but my object has not been to find how nearly it was possible to approximate to observation, but how nearly the deductions, legitimately arising out of our first hypothesis, corresponded with the same. Churchman, in his Magnetic Atlas, has assumed a certain distance and movement of rotation, which give nearer approximations than those found above, but they are dependent upon no previous principle, and are inconsistent with every other magnetic law; whereas our distance and motion are drawn immediately from an independent hypothesis, and are perfectly consistent with every known principle of terrestrial magnetism.

PART III.

On Electro Magnetism.

SECTION I.

SKETCH OF THE PRESENT STATE OF ELECTRO MAGNETISM.

230. IT was for many years suspected that there existed a strong analogy, if not a complete identity, between the electric and magnetic fluids, and various attempts were made to establish such relation on satisfactory principles. It was known, for instance, that lightning destroyed and reversed the polarity of magnetized needles, and that it produced a magnetic power in pieces of steel which had not before any such action. Now lightning and electricity have been long known to be identical; consequently, electricity ought to produce similar effects to lightning on magnetic and simple steel bars; but the attempts which were made to discover a satisfactory proof of this action by means of the electric apparatus were not attended with success; at least all that was effected in this way amounted only to communicating the

magnetic property to steel bars, but without the experimenter being able to predict in what directions the poles would lie, and therefore was little more than might be produced by a blow, by twisting, and various other means. It was indeed stated that the magnetism was more fully developed when the shock was passed through the needle transversely, than when it passed lengthwise; but still no definite conclusions could be drawn from the experiments.

231. Philosophers having thus failed of tracing the analogy between the electric and magnetic fluids, by means of the electrical apparatus, had next recourse to the Galvanic battery, which was known to possess electrical properties. Of these experiments those of Ritter are the only ones of any importance. He stated that he had succeeded, by placing a Louis d'or in contact with the extremities of a galvanic circuit, in giving to it a positive and negative electric pole, which remained after it had been in contact with other metals; he also magnetised a gold needle by means of the galvanic battery, and seems to have had some obscure ideas of electric terrestrial poles at right angles to the magnetic poles. These experiments, however, were never much regarded, and the relation between the two fluids seemed still to remain doubtful.

232. Soon after the time that Ritter made his

experiments, Professor Œrsted, of Copenhagen, published a work in which some hints are thrown out respecting the analogy between the electric, galvanic, and magnetic fluids; which were supposed to differ from each other only in their degree of tension. The galvanic fluid is there conceived to be more latent than the electric, and the magnetic still more so than the galvanic. The science, however, made no farther progress from this time (1807) till the year 1820, when the same learned Dane succeeded in establishing the reciprocal action of the galvanic and magnetic fluids upon each other by the most satisfactory experiments. These have been since repeated, and much extended by Ampere, Biot, Arago, in France; by Sir H. Davy, Professor Cummings, and Mr. Faraday, in England, and have thus led to the establishment of a new branch of philosophy designated electro-magnetism, of which it is proposed to give a concise view in the following pages.

233. *Œrsted's experiments.*—As these leading experiments are very concisely and clearly stated by the author, we shall give them in his own words.

The galvanic machine being charged, and its poles connected by a wire of any metal (which may be called the *conductor* or *uniting wire)*, the following effects will be noticed:

"Let the straight part of this wire be placed

horizontally above the magnetic needle properly suspended, and parallel to it. If necessary, the uniting wire is bent so as to assume a proper position for the experiment. Things being in this state the needle will be moved, and the end of it next the negative side of the battery will go westward.

"If the distance of the uniting wire does not exceed three quarters of an inch from the needle, the declination of the needle makes an angle of about 45°. If the distance is increased, the angle diminishes proportionally. The declination likewise varies with the power of the battery.

"The uniting wire may change its place, either towards the east or west, provided it continue parallel to the needle, without any other change of the effect than in respect to its quantity. Hence the effect cannot be ascribed to attraction; for the same pole of the magnetic needle which approaches the uniting wire, while placed on its east side, ought to recede from it when on the west side, if these declinations depended on attraction and repulsions. The uniting conductor may consist of several wires or metallic ribbons connected together. The nature of the metal does not alter the effect, but merely the quantity. Wires of platinum, gold, silver, brass, iron, ribbons of lead and tin, a mass of mercury, were employed with equal success. The conductor does not lose its

effect though interrupted by water, unless the interruption amounts to several inches in length.

"The effect of the uniting wire passes to the needle through glass, metals, wood, water, resin, stone ware, stones, for it is not taken away by interposing plates of glass, metal, or wood. Even glass, metal, and wood, interposed at once, do not destroy, and indeed scarcely diminish the effect. The disc of the electrophorus, plates of porphyry, a stone-ware vessel, even filled with water, were interposed with the same result. We found the effects unchanged when the needle was included in a brass box filled with water. It is needless to observe that the transmission of effects through all these matters has never before been observed in electricity and galvanism. If the uniting wire be placed in a horizontal plane under the magnetic needle, all the effects are the same as when it is above the needle, only they are in opposite directions; for the pole of the magnetic needle next the negative end of the battery declines to the east.

"That these facts may be more easily retained, we may use this formula,—the pole above which the negative electricity enters is turned to the *west*; under which, to the *east*.

"If the uniting wire be so turned in a horizontal plane as to form a gradually increasing angle with the magnetic meridian, the declination of the needle *increases*, if the motion of the wire be

towards the place of the disturbed needle; but it diminishes if the wire moves further from that place.

"When the uniting wire is situated in the same horizontal plane in which the needle moves, and parallel to it, no declination is produced either to the east or west; but an *inclination* takes place, so that the pole next which the negative electricity enters the wire is *depressed* when the wire is situated on the west side, and elevated when situated on the east side.

"If the uniting wire be placed perpendicularly to the plane of the magnetic meridian, whether above or below it, the needle remains at rest, unless it be very near the pole; in that case the pole is *elevated* when the entrance is from the *west* side of the wire, and depressed when from the east side.

"When the uniting wire is placed perpendicularly opposite to the pole of the magnetic needle, and the upper extremity of the wire receives the negative electricity, the pole is moved towards the east; but when the wire is opposite to a point between the pole and the middle of the needle, the pole is moved towards the west. When the upper end of the wire receives positive electricity, the phenomena are reversed.

"If the uniting wire be bent so as to form two legs parallel to each other, it repels or attracts the

magnetic poles according to the different conditions of the case. Suppose the wire placed opposite to either pole of the needle, so that the plane of the parallel legs is perpendicular to the magnetic meridian, and let the eastern leg be united with the negative end, the western leg with the positive end of the battery, and in that case the nearest pole will be repelled either to the east or west, according to the position of the plane of the leg. The eastmost leg being united with the positive, and westward with the negative side of the battery, the nearest pole will be attracted. When the plane of the legs is placed perpendicular to the place between the pole and the middle of the needle, the same effects recur, but reversed.

"A brass needle, suspended like a magnetic needle, is not moved by the effect of the uniting wire. Needles of glass and of gum lac, remain likewise quiescent."

233. These facts having laid the foundation of the present interesting science of electro magnetism, I have thought it best to give the statement in the author's own words; but in what follows, it will be necessary to be more concise.

The experiments of Mr. Œrsted were no sooner promulgated, than they were repeated and considerably extended by M. M. Ampere, Arago, and Biot; by Sir H. Davy, Mr. Faraday, and Professor Cummings, as well as by several celebrated German

philosophers; and many curious and interesting facts and phenomenas were thus elicited.

234. M. Ampère, for instance, discovered that not only there is a reciprocal action between the galvanic wire and the magnetic needle, but that two such wires act upon each other, by attraction, when they both proceed from the same extremity of the battery, and by repulsion when they proceed from opposite extremities; that is, two conducting wires, free to move, being placed parallel to each other, and the corresponding extremities proceeding to the like poles of two different galvanic machines, the wires will be attracted to each other; but if the corresponding extremities of the wire proceed from contrary poles of the batteries, then the wires will indicate a mutual repulsion between them.

235. Again, it was shown by M. Arago that the connecting wire of a galvanic battery had an obvious action upon iron filings, and that it would hold them suspended like an artificial magnet, but that they fell the moment the contact with the battery was broken. The same thing was discovered by Sir H. Davy, who also showed that the filings on the opposite sides of two parallel wires attracted each other, and that those on the same sides repelled.

236. The latter experiments naturally led to an attempt to magnetize steel wires by the galvanic

battery, in which the first successful attempt was made by Sir H. Davy, although it was effected at nearly the same time by M. Arago. In the first instance the needle was simply laid transverse of the single wire, and the operation required a certain time; but M. Arago afterwards made use of a spiral wire, and was thus enabled to produce the maximum effect almost instantaneously. Sir H. Davy also succeeded in magnetizing steel needles with the electrical battery at very considerable distances, and thus demonstrated that the magnetic power was not peculiar to the galvanic apparatus.

237. The next question was, since there is so obvious a connection between the freely suspended galvanic wire and a magnet, has the former a directive quality from the influence of the terrestrial magnetism?

This led M. Ampere to the construction of a simple apparatus, which will be described in a subsequent section, and by which he proved that if a part of the galvanic wire, bent into the form of a rectangle nearly shut, and free to move, be left to the action of the terrestrial magnetism, it will adjust its plane to one perpendicular to the magnetic meridian, and that by giving to a similarly formed wire a freedom of motion on a horizontal axis, it will conform itself to that plane which in our first part has been called the plane of no at-

traction; that is, the plane of the wire will in all cases have a tendency to place itself at right angles with the plane of the magnetic meridian, and to the line of direction of the dipping needle.

These experiments are more fully illustrated in our last section.

238. At this stage of the enquiry Mr. Faraday, of the Royal Institution, commenced his enquiries. He proved that the action which had hitherto been noticed between the magnetic and the galvanic wire, was neither attraction nor repulsion, but was of such a nature as to give to the magnetic needle a tendency to revolve about the wire, and he at length succeeded in producing this rotation; viz. he was enabled by a very simple apparatus, which we have described in our third section, to cause either pole of a magnet to revolve about a fixed galvanic wire, and conversely, by fixing the magnet, he caused the wire to revolve about the former, and by the same apparatus also, the wire and magnet being both free, may be made to revolve about each other; and he subsequently was enabled to produce a rotation of the wire by the mere influence of the terrestrial magnetism upon it. These beautiful experiments threw an entire new light upon the science of electro magnetism.

239. M. Ampere having been informed of Mr. Faraday's experiments, succeeded in causing the magnet to revolve on its own axis, by introducing

it as a part of the galvanic circuit; an experiment attempted by Mr. Faraday, but which he had not been able to perform; and Sir H. Davy by his experiments on the mercurial vortices, proved also the rotation of the wire on its axis, which is effected in another manner in our 10th experiment. See section iii.

240. Such was the state of this science when I undertook the experiments reported in the following section, and by which, if I have not deceived myself, the whole of the apparently anomalous actions hitherto observed, may not only be explained, as to the general effects, but the disturbance on the needle computed for any determinate position of the compass and wire, in a manner very similar, but more simple, than that which has been illustrated in reference to the iron ball and magnetic needle.

It may be proper to observe that several experiments, besides those alluded to above, had already been made by other philosophers, and which led to many curious facts, but as they do not appear to have had any influence in advancing the theory of the science they have not been referred to in the preceding sketch; but some of them are given in our third section.

SECTION II.

ON THE MATHEMATICAL LAWS OF ELECTRO MAGNETISM.*

241. All the experiments that have been made on the subject of electro magnetism, since the first discovery of that power by Mr. Œrsted, seem to indicate a strong affinity, although not a complete identity, between the simply magnetic and the electro magnetic fluids; or, if the identity be admitted, still a certain difference must be conceived to have place in the modes of action.

In the preceding parts of this work I have attempted to reduce the laws of induced magnetism to mathematical principles, and to render the results susceptible of numerical computation, the mass of iron, and its position with respect to the compass, being given; and as soon as I heard of Mr. Œrsted's discovery, I was desirous to establish, on similar principles, the law of electro magnetism; but it was some time before I was able to construct an apparatus convenient for the purpose. Having, however, at length effected this necessary prelimi-

* The substance of this section was placed in the hands of Sir H. Davy by Major Colby, last March, and was read before the Royal Society, May 23. I am sorry I have been obliged to publish it before the council has decided respecting its appearance in the Transactions.

nary to my satisfaction, I proceeded to make the course of experiments, and to undertake the investigations which form the subject of the present section.

242. My first object was to repeat very carefully all the experiments of Mr. Œrsted, M. M. Ampere, and Arago; of Sir H. Davy and Mr. Faraday, with some others suggested by the results thus obtained; and having attentively considered all the peculiarities of action thus developed, I was led to consider that all the apparently anomalous effects produced on a magnetized needle by the action of a galvanic wire, might be explained by the admission of one simple principle; viz. *that every particle of the galvanic fluid in the conducting wire acts on every particle of the magnetic fluid in a magnetized needle, with a force varying inversely as the square of the distance; but that the action of the particles of the fluid in the wire is neither to attract nor to repel either poles of a magnetic particle, but a tangential force which has a tendency to place the poles of either fluids at right angles to those of the other; whereby a magnetic particle, supposing it under the influence of the wire only, would always place itself at right angles to the line let fall from it perpendicular to the wire, and to the direction of the wire itself at that point.*

I pretend not to illustrate the mechanical princi-

ples by which such an action can be produced; I propose only to show, that if such a force be admitted, all the results obtained from the reciprocal action of a galvanic wire and a magnetized needle may not only be explained, but computed, and that the results agree numerically with experiments.

243. The galvanic machine which I have employed, is constructed after the principle of Dr. Hare's colorimoter, differing from his only in the mechanical contrivance for lowering and raising it out of the fluid; it consists of 20 zinc and 20 copper plates, each ten inches square; but it possesses a power far beyond what is requisite for repeating all the experiments alluded to in the commencement of this paper.

244. That part of the apparatus which peculiarly appertains to the experiments I am about to detail, is represented in (fig. 1. pl. 4). A B is an upright stand, placed near the poles of the battery; *a b*, *c d*, are two staples of stout copper wire, driven into the upright, the two ends at *b* and *c* passing quite through, as shown at C and Z; and on which two wires are fastened by spiral turns, and with which the communication is made with the poles of the battery; *e f*, *g h*, are two copper wires of the same dimension as the staples, each four feet long, having their ends flattened and drilled so as just to enable them to slide freely upon the wires *a b*, *c d*, and the vertical wire *f h*, also 4 feet

in length, which passes through a hole in the top of the table F G H I, and so tight as to render it perfectly fixed. On the plane of the table, which is two feet in square, the circle N E S W is described about the centre *o*, and divided into the points of the compass and smaller divisions ; N S, is an index or box ruler, through which the wire *f h* passes, so that the former may be turned freely about the latter, and set to any proposed azimuth. On this ruler is placed the small compass *c'*, by means of which the deviation at any time may be taken ; *c''* is another compass placed on the top of the support L *c''*, and is intended to remain fixed in its place, in order to serve as a standard for estimating and comparing the power of the battery at different times.

For the principal experiments this apparatus is placed so that the plane of the rectangle of wires is perpendicular to the magnetic meridian ; because in this position the horizontal wires being east and west, they have no effect in deflection the needle from its direction, (at least there is only one exception to this, which will be noticed hereafter,) and consequently all the effect produced upon the needle during the rotation of the index in the circle N E S W, is due to the vertical wire only, except so far as the horizontal wires may increase or diminish the directive power of the needle.

This, however, in the cases to which we shall refer is very inconsiderable.

245. But in order that we may know precisely what part of the change of deviation between one situation of the compass and another is actually due to that change of position, recourse must be had to the standard compass, which, always remaining fixed in its position, may be used as a constant indicator of the strength of the battery. But as the application of this measure to computation is involved in principles not at present explained, it will be proper first to inform the reader of the means which I employ in the first instance to preserve an uniformity of action during every separate course of experiments. These were as follow:—

246. The vessel which contains the dilute acid, into which the plates are immersed, holds nearly twenty gallons; and I begin the experiments with little more than twelve gallons; moreover the plates are not, in the first instance, let down to their lowest point. The intensity shown by the standard compass after the connection has been made, some minutes is noted; and by breaking off and making the contact anew, this same intensity occurs again, the power being always strongest when the contact is first made; then when the standard compass returns to its former

bearing, the observation with the other compass is taken; the contact broken, and renewed, and so on as long as the battery retains sufficient power. When this fails, the plates are lowered a little more, the power thus increased, and the observations resumed, till at length the plates being wholly down, and the power too weak, recourse is had to a supply of more dilute acid; by which means a tolerably steady action is kept up longer than is necessary for any series of experiments of this kind. It will be observed here, that in this case the only use made of the standard compass is to indicate the *same intensity of action*, and consequently involves no theoretical principle that will be objected to by the most scrupulous theorist or observer, but it will be seen in a subsequent article that this indicator is susceptible of a more extensive application.

247. Having thus made the reader acquainted with the means employed and the precautions adopted, to ensure accuracy, I shall proceed now to explain the principles of computation, and to compare the numerical results thus obtained, with those derived from experiments.

According to the hypothesis (art. 242) if we conceive the wire in the first instance to be vertical, and the compass placed to the north or south of it, and opposite its middle point, the centre of

action will lie in the horizontal plane, and at right angles to the natural horizontal direction of the needle. The latter, therefore, (which for simplicity sake we shall at present consider as indefinitely short with regard to the distance), will at either of those points, be acted upon by two rectangular forces; viz. the galvanic force in an east and west direction, and which we may denote by f, and the natural magnetic or directive force m; consequently, according to the principle of forces, the resultant will be expressed by $\sqrt{(f^2 + m^2)}$ and the angle which it makes with the natural direction of the needle, being called Δ, we shall have

$$\tan \Delta = \frac{f}{m} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Hence the magnetic force being constant, the tangent of the needle's deviation at the north or south will be a correct measure of the galvanic power.

248. We have thus a principle by means of which we may verify a part at least of our theory by experiments.

For example; since by the supposition every particle of the galvanic vertical wire acts inversely as the square of its distance from a given point, we ought to find a determined relation between the tangent of deviation, and the length of the wire; or the length of the wire remaining constant,

between the tangent of deviation and the distance, provided always that the intensity of the battery remain constant.

The apparatus already explained furnishes us with the opportunity of making both these comparisons. For by means of the sliding horizontal rods, the vertical conducting part of the wire may be shortened in an instant; and in the second case, it is only necessary to slide up the compass to different distances, which may likewise be done so quickly, that it will not be necessary even to have recourse to the standard compass.

It is fortunate also that the calculation here alluded to is of the simplest kind. For denoting the length of the wire by 2 l, and the distance of the compass by d; assuming also x as any variable length, the corresponding elementary action at this distance will be $\frac{\dot{x}}{d^2 + x^2}$, and the sum of these actions will be

$$\int \frac{\dot{x}}{d^2 + x^2} = \frac{1}{d} \text{ arc. tan } \frac{x}{d}$$

which vanishes when x vanishes; and which therefore when $x = l$, and the two lengths are included, becomes

$$\frac{2}{d} \text{ arc. tan } \frac{l}{d}$$

consequently if we denote the deviation, as we have done above by Δ, we ought to find this force vary inversely as tan Δ, or

$$\cot \Delta \left\{ \frac{1}{d} \text{ arc. tan } \frac{l}{d} \right\} = \text{a constant quantity.}$$

The following are a few out of numerous experiments of this kind which I have made, and which have been all found equally satisfactory.

249. *Experiments to determine the magnetic deviation caused by a galvanic vertical wire at different distances. Length of vertical wire* 36 *inches.*

Deviation by standard compass.	Distance of the other compass from the wire.	Mean* observed deviation. Δ	Value of $\frac{2}{d}$ arc. tan $\frac{l}{d}$ = A	Constant product. A cot Δ
° ′		° ′		
25 0	12 inches	5 37	18·772	190880
Ditto	8 ditto	11 15	34·100	171432
Ditto	6 ditto	16 30	47·712	161062
Ditto	4 ditto	26 30	77·500	154440
			Mean	164728

250. When it is considered that these observations were made on a compass needle only one inch in length, and that the divisions extended only to quarter points, it is impossible to expect a closer approximation. The needle and card, however, being delicately suspended, and the latter very distinctly divided, I could depend upon my observations to the nearest *degree*; for by means of a strong magnifying power I could always bisect and trisect the quarter points without any very sensible error.

* That is, the mean of two observations at each station of the compass; the contact being changed. The same is to be understood of the deviation with the standard compass.

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251. *To determine the magnetic deviation caused by a vertical galvanic wire; the length being varied, but the distance constantly* 9 *inches:*

Deviation by standard compass.	Length of vertical wire.	Observed deviation. $= \Delta$	Value of $\frac{2}{d}$ arc. tan $\frac{l}{d}$ $= A$	Constant product. $A \cot \Delta$
		° ′		
——*	36 inches	22 30	63·450	15318
——	24 ditto	18 16	53·133	16097
——	16 ditto	12 0	41·633	19557
——	12 ditto	8 25	33·683	22764
			Mean	18220

252. These results (except the last) although not so uniform as the above will be found, notwithstanding as nearly so as we have any reason to expect, particularly as we were not able in these to avail ourselves of the use of the standard compass.

I am, however, inclined to attribute the discrepance between the observed deviation and the computed, as the vertical wire shortens, to the approach

* The standard compass cannot be used in these experiments, because the wire by which it is deflected is necessarily shortened with that on which the observations are made.

of the horizontal wire, which has a tendency to increase or decrease the directive power of the needle, according to the pole with which the wire is connected, (as will be seen as we proceed) and thereby rendering the action of the vertical wire more or less effective, according to the circumstances of the connection. (See art. 262.)

253. Having thus far versified our hypothesis by experiment, let us now proceed to the consideration of the deviation in different azimuths.

Let Z (fig. 2.) represent the horizontal section of a vertical wire proceeding from the zinc end of the battery downwards, o a particle of the magnetic fluid whose natural direction is in $n\ s$, join $Z\ o$, and draw $r\ t$, perpendicular to $Z\ o$; then, according to the hypothesis, the direction of the force excited by the wire Z, will be in the line $r\ t$. Now the intensity of this force to turn the particle about o, will vary as $\sin \angle\ t\ o\ n$, or as $\cos S\ Z\ o$, and its intensity in the line $n\ s$, will vary as $\sin S\ Z\ o$, which latter force will be additive to the directive power of the terrestrial magnetism. Let the latter force on the horizontal needle be called m, and the galvanic force in $r\ t = f$, also the angle $S\ Z\ o = \phi$, S being the south point of the horizon.

Then the particle o, will be urged by the two rectangular forces.

$m + f \sin \phi$ in the direction $n\ s$

$f \cos \phi$ in the direction perpendicular to $n\ s$, consequently, denoting the angle of the resultant, or the deviation of the particle from the line $n\ s$ by δ, we shall have from the known principle of forces

$$\tan \delta = \frac{f \cos \phi}{m + f \sin \phi} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Let Δ denote the deviation of the needle at the south point; then, from what has been already demonstrated (equation i)

$$f = m \tan \Delta$$

which being substituted for f in the above equation, reduces it immediately to

$$\tan \delta = \frac{\cos \phi}{\cot \Delta + \sin \phi} \quad \ldots\ldots\ldots\ldots \quad (3)$$

From which equation (the deviation Δ being supposed known) the deviation δ at every other azimuth may be computed.

254. This formula is as comprehensive as it is simple, and indicates by the changes of the signs in $\sin \phi$ and $\cos \phi$, a variety of cases, the whole of which I have most satisfactorily confirmed by experiments. These may be stated as follow:

First, $\cot \Delta$ may be greater, equal to, or less than unity, accordingly as the observed deviation at the south, is less, equal to, or greater than four points, or 45°. This consideration leads to three distinct cases.

CASE I. *when* cot $\Delta > 1$.

Here the denominator of the formula is necessarily positive throughout the circle. In the first quadrant of which, sin ϕ, and cos ϕ, being both positive, tan δ is also positive, and the deviation is all one way.

2. When $\phi = 90°$, cos $\phi = 0$; and tan $\delta = 0$; there is therefore no deviation at the east point.

3. In the second quadrant, cos ϕ is negative, as is also tan δ; the deviation is therefore now the contrary way, but it is the same in quantity in all equidistant situations north or south of the east.

4. At the north point, sin $\phi = 0$, cos $\phi = -1$, and we have

$$\tan \delta = -\tan \Delta$$

the deviation is therefore the same as at the south, but in an opposite direction.

5. In the third quadrant, cos ϕ is still negative, as in the second, but sin ϕ is also negative, and therefore the deviation although of the same kind in direction as in the second quadrant, is greater in its amount, the denominator being less.

6. At the west, the cos ϕ vanishes, tan δ becomes zero, and the needle again resumes its natural direction.

7. In the fourth quadrant, cos ϕ again becomes positive, the deviation changes in its quality, but is the same in quantity as in the third quadrant.

CASE II. *when* cot $\Delta = 1$.

8. Here the results are precisely the same in the four quadrants with respect to direction, as in those above explained; except that at the west point, where sin ϕ and cot Δ, being each equal to unity, and with contrary signs, the denominator vanishes with the numerator, and the needle is indifferent to any direction.

CASE III. *when* cot $\Delta < 1$.

9. Here in the first two quadrants the deviation has the same character as in the preceding cases. But in the third quadrant, the denominator of the fraction vanishes before the needle reaches the west point, tan δ becomes infinite, and the deviation is 90°; that is, the needle will stand east and west.

10. For the remainder of this quadrant, tan δ is *plus*, and the character of the deviation changes, till at the west point the needle is found inverted.

11. From this point cos ϕ becomes positive, but the denominator being negative, tan δ is negative, and remains so till it becomes infinitely negative, as on the other side of the west, and the deviation is 270°.

12. Lastly, from this point to the south, the denominator is positive, and tan δ has the same sign as at first, and at the south point resumes its original deviation, provided the intensity of the battery has been preserved constant.

255. To illustrate this last case by an example, let us suppose that the deviation at the south point is greater than 45°, as for instance 50°, then since cotan 50° = 0·83909, and sin (180° + 57° 2′) = — 0·83914; the denominator will vanish when the compass is placed 57° from the north towards the west; the tan δ is therefore infinite, or the needle will at this place stand east and west.

Proceeding on towards the west, the deviation will increase more and more till, at the west point itself, the needle will be found inverted. At 57° from the south, or 33° from the west towards the south, the denominator again vanishes, and the needle stands west and east; from which position the deviation decreases till it becomes 50° again at the south point as at first.

Hence it appears that in passing the index which carries the compass from the position west 33° N to west 33° S, that is through 66° only, the needle ought to make a complete semi-revolution on its pivot; whereas if we pass the index the other way, viz. through the north, east and south, we must move it through 294° to produce the same motion in the needle. A single trial will show how correctly this theoretical deduction accords with experiment.

256. In the above case the needle makes a complete revolution on its pivot while it is carried round the wire; but this will not happen if the deviation

at the south be less than 45°. Let us, for example, suppose it to be 40°; then cotan 40° = 1·19175, and sin ϕ is never greater than + 1, or less than — 1; consequently, the denominator will not become zero. In this case the deviation will be the greatest when

$$\frac{\cos \phi}{\cot 40^\circ + \sin \phi} \text{ is a maximum,}$$

which happens when sin ϕ = — tan 40°, viz. at 33° from the west towards the north and south; but in passing the index through this arc, the north point of the needle will not, as in the former instance, pass through the south, but will fall back towards the north, passing through it as the index passes through the west. Here again the theory is most satisfactorily confirmed by observation.

As any one may repeat these experiments, and make his own remarks, I shall not insist farther upon them in this place.

It is proper, however, to caution the reader that to ensure success, it is necessary to have a short needle, and to work at as great a distance from the wire as the power of the battery will allow of; because the above deductions have been made by supposing the length of the needle inconsiderable in comparison with the distance and length of the wire.

257. The following is one series of numerical results derived from the preceding formula, with the corresponding observations.

EXPERIMENTS

On the deviation of the needle caused by a vertical galvani wire at different azimuths, the deviation at the south poin being 16° 30′, *and the standard compass showing alway* 25°.

Azimuths.	Value of $\tan \delta = \frac{\cos \phi}{\cot. \Delta + \sin \phi}$	Corresponding angle of deviation.		Observed deviation.		The same observed points and quarter points.	
		°	′	°	′	p.	qp.
South E	+ ·296	+16	30	+16	30		
S 2 points E	+ ·245	+13	46	+14	4	1	1
S 4 points E	+ ·173	+ 9	49	+ 8	26	0	3
S 6 points E	+ ·089	+ 5	6	+ 5	37	0	2
East	000	0	0	0	0	0	0
N 6 points E	— ·089	— 5	6	— 5	37	0	2
N 4 points E	— ·173	— 9	49	— 8	26	0	3
N 2 points E	— ·245	—13	46	—14	4	1	1
North	— ·296	—16	30	—16	47	1	2
N 2 points W	— ·389	—21	16	—22	30	2	0
N 4 points W	— ·265	—14	51	—14	4	1	1
N 6 points W	— ·156	— 8	53	— 8	26	0	3
West	000	0	0	0	0	0	0
S 6 points W	+ ·156	+ 8	53	+ 8	26	0	3
S 4 points W	+ ·265	+14	51	+14	4	0	3
S 2 points W	+ ·389	+21	16	+22	30	2	0
South	+ ·296	+16	30	+16	47	1	2

Although the aberrations in these results are greater than could be admitted in experiments which allowed of more accurate means of observation, yet they are such as may, I trust with confidence, be adduced as a confirmation of the hypothesis that has been advanced. They will probably

be repeated with more accurate means than I possess, and on a larger scale; when a closer approximation will, I have no doubt, be obtained.

258. It may be proper here to observe, that the sign of plus or minus prefixed to the angle of deviation, is wholly arbitrary. I have called it *plus* when the deviation is easterly, and *minus* when it is westerly.

This sign however being thus fixed, it is necessary to give an indication of the course of the needle as it is affeced by the galvanic wire, which at present has only been stated in general terms; viz. that it has a tendency to arrange itself at right angles to the line joining the nearest point of the wire and its axis.

To conceive this effect more particularly, the reader must consider himself as a part of the galvanic circuit, having his head towards the zinc end of the battery, and his face to the needle; then the effect will be to carry the north end of the needle placed before him always to his left hand.*

This is in all cases sufficient to remember, because it necessarily implies that the south end is carried to the right, and that if the wire proceed from the other extremity of the battery, his direction will be reversed, as will also the motion of the needle, and the signs of the angles of deviation.

* This supposes a simple combination of two plates; it is the reverse with a compound battery.

259. We have at present shown no other application of the standard compass than that of its indicating an uniformity of power in the battery at the time of registering the observation; it may, however, as we have already observed, be equally useful in other cases. For example, let us suppose it to be placed (as in the experiments reported above) to the north of the wire, and let its deviation at any given intensity of the battery be D, while that of the other compass at the north or south is Δ, and let its deviation with a different intensity be D′, and the corresponding deviation with the other compass be Δ'; then it is obvious from what has been stated, that

$$\tan D : \tan \Delta :: \tan D' : \tan \Delta'$$

consequently, if the power of the battery between any two observations is such as to alter the deviation of the standard compass from D to D′, that of the principal compass will be found from the equation

$$\tan \Delta' = \frac{\tan \Delta \tan D'}{\tan D}$$

We have therefore only to introduce this value of $\tan \Delta'$ into our general equation,

$$\tan \delta = \frac{\cos \phi}{\cot \Delta' + \sin \phi}$$

which will thus become

$$\tan \delta = \frac{\cos \phi}{\dfrac{\tan D}{\tan \Delta \tan D'} + \sin \phi}$$

a formula which is applicable to all degrees of intensity.

260. Let us now examine the circumstances attending the deviations caused by a horizonal wire placed in the magnetic meridian.

In this case conceive S E N W (fig. 3) to represent a vertical circle in the plane of the section of the wire, and corresponding with its middle point, E and W being its east and west points. Let o be a magnetic particle in a horizontal needle, the direction of which is perpendicular to the plane S E N W. Let the force in the line $r\ t$ be denoted by f as before, and call the angle S Z $o = \phi'$. Resolve f into the two rectangle forces, $f \sin \phi'$, $f \cos \phi'$; the former of which being perpendicular to the horizon will only affect the inclination of the needle; but the other force, $f \cos \phi'$, being horizontal, and in an east and west plane, will be wholly effective in producing its deviation.

Let Δ be the deviation at S, which will be the same whether the wire be horizontal or vertical, because in both cases the tangential force is horizontal and perpendicular to the needle.

Consequently, as in the former case,

$$f = m \tan \Delta$$

and our two forces become

$m \tan \Delta \cos \phi'$, in the horizontal plane

$m \tan \Delta \sin \phi'$, in the vertical plane

the former, as we have seen, is the only one which

affects the bearing of the needle, and is therefore the only one we have to examine.

From this we obtain,

$$\tan \delta = \frac{m \tan \Delta \cos \phi'}{m} = \tan \Delta \cos \phi'$$

and hence we learn, that as the compass is carried round the wire in a vertical circle, the tangent of the deviation of the needle will vary as the cosine of the angle S Z o.

261. This cosine being zero at the east point,* the tangent δ vanishes and the needle stands in its natural direction, but will be inclined downwards by the force

$$\tan \Delta \sin \phi'$$

Beyond the east point, $\cos \phi'$ becomes negative, the sign of $\tan \delta$ changes, and consequently the deviation is now the contrary way. At N, $\cos \phi = -1$ and $\tan \delta = -\tan \Delta$, we have therefore here the same deviation as at first, but in an opposite direction.

In the next or third quadrant, $\cos \phi'$ is still negative, and the deviation is the same both in quantity and direction as in the second quadrant. At the west point, $\cos \phi'$ again vanishes, and the needle returns to its proper direction. In the

* By the east point is meant that point in the circle which is to the east of the wire, and in the same horizontal plane with it.

fourth quadrant cos ϕ' is positive, and the deviation is the same both in quantity and direction as in the first quadrant. There is not therefore in this case the same kind of anomalous deviation which takes place in the vertical wire.

The other force tan Δ sin ϕ, which affects the needle's inclination, is greater at the east and west points; it is nothing in the zenith and nadir, and in all intermediate positions it varies as the sin ϕ. All these deductions are perfectly consistent with the general character of the observations of Mr. Œrsted, and with my own, and I have not therefore thought it requisite to submit them to the test of numerical experiments.

262. It has been said by some observers that a horizontal wire arranged east and west has no power on the needle, except to disturb its inclination. But it ought obviously, according to our theory, to produce the same anomalous action as the vertical wire in the case where cot Δ is equal to or less than unity; because then the galvanic force being equal to, or exceeding the terrestrial directive force, it ought, when the two are opposite, to reverse the direction of the needle; and this will be found to be the case by applying the latter above the upper, or below the lower horizontal wire, when the former is connected with the zinc end of the battery, and the reverse with the opposite connection. It is this effect to weaken or

reverse the direction of the needle that we have alluded to in (arts. 250 and 252.)

263. I might now proceed to a variety of other investigations for different directions of the wire, or even generally for every possible direction, and for a needle freely suspended and susceptible of motion in all directions; but as it would be difficult to submit the results to the test of numerical experiments, I leave the task to those who have more leisure for pursuing the subject, and who may perhaps be disposed to enter upon the investigation in more general terms. My results are necessarily only approximative; because I have throughout supposed the needle indefinitely short in comparison with the distance and length of the wire; but by this means I have rendered the subject perfectly intelligible to every one; whereas had I taken the actual case of the reciprocal action of every particle of the fluid in the wire upon every particle in the needle, and had been able to complete the investigation, it could only have been understood by a few mathematicians; at the same time the minute corrections thus introduced would not have been appreciable in the comparison of the results with experiments; these latter being necessarily both liable to small irregularities and difficult to observe.

264. It will have been noticed that I have only attempted to illustrate the nature of the action which has place between a galvanic wire and the

compass, and not that of one galvanic wire on another. What modification the hypothesis may require to explain the latter class of phenomena, will be examined hereafter. I have hitherto supposed only one species of action in the galvanic wire; but it is highly probable that it is compound, and that while the north end of the needle is carried in one direction, by the action we have supposed, the south end is carried in an opposite direction; not merely as a consequence of the first force, but by a distinct power. This will not, however, in any respect affect our investigation; because both forces lead to similar results.

We have seen a precisely analogous instance in our investigation of the laws of induced magnetism, (art. 158); where it appears that we obtain exactly the same results, whether we consider the magnetic fluid as simple, and acting equally on each extremity of the needle, or as compound, and acting reciprocally on both; and it was only for the sake of certain analogies I was desirous of preserving, that I was induced to adopt the latter hypothesis. Similar reasons may also render it necessary, in this case, to admit the existence of a compound action in the galvanic wire; but which, as we have already stated, will in no respect affect the preceding investigations.

I am well aware of the difficulty of conceiving the mechanical principles by which such a tan-

gential force, as is here assumed, can operate; but on the other hand it must, I think, be conceded, that the simple power of attraction is equally difficult to conceive, and that we admit it, not from having any idea of the *modus operandi*, but because we find that it leads to results that are consistent with actual observations; and I have endeavoured to show, in the preceding pages, that the force we have assumed is admissible upon precisely the same ground.

Let us now see how far the same hypothesis is consistent with the various other facts and phenomena that have been elicited by different philosophers in their pursuit of this interesting inquiry.

SECTION III.

A COURSE OF ELECTRO MAGNETIC EXPERIMENTS.

265. In the preceding sections of this part, I have endeavoured first, to give a concise sketch of what has been effected in this science since its first introduction by Mr. Œrsted; and secondly, to illustrate the theoretical principles on which we have supposed the action to depend. I have also proved, by a comparison of several numerical results, that the theory assumed is consistent with experiments

in the particular cases in question; but it still remains to be shown that it is likewise consistent with the various facts and phenomena that have been elicited by the several experimenters to whom we have already referred.

In following up this view of the subject I shall no longer regard the order in which the several isolated facts have been developed, but shall endeavour rather to be guided by that of their natural dependence on each other, in every case, however, attributing to their proper author the experiments which are due to his ingenuity.

In such a course of experiments as is here proposed, I ought first to commence by showing the action of the galvanic wire on the compass needle, and to elucidate the several peculiarities of action observed by Mr. Œrsted; but as this has been so fully entered upon in the preceding section, I shall content myself with referring the reader to that part, for an explanation of every fact hitherto known of the reciprocal action of a galvanic wire and a magnetic needle, and proceed to the next course of experiments, which have no reference to the deviation of the needle suspended as such, but simply to the reciprocal action of a magnetic bar and the galvanic wire.

Experiment I.

To magnetize steel bars with the galvanic battery.

266. Take a piece of steel wire, as for example, a sewing needle, and dip its ends first into steel or iron filings, in order to ascertain that it has no magnetism already in it, which will be the case if the particles of iron do not adhere to it; if they do, another needle must be tried, till we find one free from every species of magnetic action; this being done, connect the ends of the battery by the conducting wire C Z, and place the needle N S across it (see fig. 4) drawing the latter backward and forward a few times, and it will be found to have acquired the magnetic property; for on immersing its extremities again in the filings they will be found to adhere to it, in the same manner as to a needle magnetized in the usual way.

This very interesting experiment is strictly conformable to our hypothesis; for according to this, the action of the galvanic particles in the wire, being tangential, will act upon the latent magnetic particles in the needle, in the direction of its length, and cause a displacement of them, precisely in the same manner as would be done by a magnet; and also, as in that case, the cohesive power of the

steel preventing the return of the fluids to their natural state, the needle will remain magnetic.

This experiment was performed nearly at the same time by Sir H. Davy, and M. Ampere; but Sir H. Davy also succeeded in effecting the same with the common electrical machine, and showed that the magnetism might be excited at considerable distances, and consequently not only without rubbing the needle on the wire as we have described, but even without the contact. It requires, however, to effect this at the distances here alluded to, a very powerful apparatus.

If the needle be made a part of the galvanic circuit, or if it be placed lengthwise of the wire, no perceptible permanent magnetic power will be developed, which is also consistent with the hypothesis; because in this case, the action of the wire will be transverse of the needle, which is the least favorable direction for the development of the magnetic power; the tendency of the action being to place the poles transversely instead of lengthwise.

Experiment II.

To ascertain the polarity of needles magnetized as in the last experiment.

267. The wire and needle being placed as in the last figure; that is, the needle being above the wire, and Z denoting the zinc end of a battery of two

plates only, it will be found that the extremity N will attract the south end of a compass needle, and the extremity S the north end; in short, that the north poles of the latent magnetic particles have been carried towards the left hand, and the south towards the right hand, agreeably to the principles indicated in (art. 258) of the preceding section.

Let now the needle be placed under the wire, instead of being placed over it, and in other respects the process described in the last example repeated, and it will be found that the polarity of the needle will be exactly the reverse of that in the last experiment, which ought to be the case according to the principle of the above article; because by this the north polarity is always carried to the left hand of the observer, who conceives himself to form the galvanic circuit, his head being towards the zinc end, and his face towards the magnet; for thus his position being now the reverse of what it was in the preceding experiment, the polarity ought to be the reverse also.

Experiment III.

To magnetize a needle by placing it in a spiral conducting wire.

268. Let Z C (fig. 5) represent a conducting wire bent into a spiral form, and let the needle *n s* be placed either naked in the spiral, or inclosed in a glass tube, or in a tube of any other matter; make

the connection with the battery, and in an instant it will be found that the needle *n s* has become strongly magnetic, having its poles posited, as shown in the figure, viz. having its north end towards the zinc extremity of the battery.

This is of course precisely similar to Experiment I. the only difference being, that by means of the spiral form given to the wire, the action upon the needle is repeated as many times as there are spires of the wire covered by it; the power excited is therefore proportionally stronger, and the magnetism more quickly communicated. The explanation of the effect produced is exactly the same as in the last experiment. If the direction of the contact be changed by supposing Z to communicate with the copper side of the battery, the effect will be in all respects the same, except that the polarity of the needle will be reversed. The end towards Z, in this case, becoming the south instead of the north pole.

Or, if a spiral, having its spires turned the contrary way, as shown in (fig. 6) be used, and Z be supposed to communicate with the zinc side of the battery, the polarity will also be the reverse of that in the first case; viz. the poles will have the direction marked in the figure; and if here again the contact be changed by connecting Z with the copper side, the poles will be once more inverted, and have the same direction as at first. These facts,

as we have stated above, are explained exactly in the same manner as those for the single wire.

In performing this experiment, I employed a glass tube about 5 inches in length and half an inch in diameter; and it was observed, when the needle was placed in it, so that one half of it projected beyond the end, that the moment the plates reached the acid,* the needle was drawn instantly to the middle of the tube, and while the contact was continued it was held suspended in the centre of the tube when the latter was held vertically; the suspending power of the spiral exceeding the power of gravity.

This effect is very curious, because the needle here remains suspended in the open space, directly in the axis of the tube, and not attached to either sides as in the usual cases of suspension by attraction.

Experiment IV.

To examine the effect of a spiral conducting wire on a floating magnetized needle.

269. Let a wire be wound about a glass tube of about half or three quarters inch diameter, and hang it within a basin of water, as shown in (fig. 7), so that the surface of the water rises to about the axis

* The connection of the spiral with the conducting wires is here supposed to be made before the plates are immersed in the acid.

of the bore; then having pierced a small piece of cork with a needle previously magnetized, so as just to preserve it from sinking when immersed in the basin, make the connection with the battery. The needle will instantly be agitated, and will soon arrange itself in front of the spiral in a direction parallel to its axis, and then suddenly dart into the interior of the tube with a force nearly sufficient to carry it to the other extremity; it then returns again towards the other end, and at length becomes stationary in the middle of the axis, arranging itself exactly parallel to it.

If the spirals have the direction shown in the figure, and Z communicates with the zinc side, the needle, if placed near the extremity of the tube A, will enter with its south end; if placed near the other extremity, it will enter with its north end; but if the direction of the spiral be changed, the needle will enter in both cases the reverse way, as it will also if the direction of the spires remain the same, but the contact be changed. This experiment will succeed equally well if the tube be placed upright in the water, the needle will then dive like a fish, and remain below till the contact is broken.

This entertaining and instructive experiment is due to Mr. Faraday; the explanation of it by our hypothesis is obvious, for the north pole of the particles of the needle being carried to the left of an observer conceiving himself coinciding with

the direction of the wire, and with his head towards Z, all the effects ought to take place precisely as above stated. M. Ampere had assimilated a spiral wire of this kind with an actual magnet, and Mr. Faraday instituted the above experiment to prove that there was not that identity which had been assumed; for by suspending a hollow cylindrical magnet in the same way, the needle was always attracted to the nearest extremity of its edge, and indicated no tendency to enter the tube.

Experiment V.

To show the effect produced by a galvanic wire on steel or iron filings.

270. This experiment is performed by strewing a quantity of iron dust or filings on a table, and bringing the connecting wire near to them, when the filings will immediately be affected by the action of the wire, some few flying towards it, and adhering to it as to a magnet; and if the wire be brought into actual contact with them, a very considerable quantity may be taken up by it, exactly the same as at the extremity of a bar magnet; but the moment the contact is broken the filings fall.

In order to produce the best effect in this experiment, the wire intended to be operated upon should be smaller than the conducting part of the circuit. This latter, in all cases, is the better for

being stout, at least $\frac{3}{10}$ of an inch in diameter; but in this, as in several other experiments, it is best to have the extremities of the wires terminated by a much smaller wire, wound round the former as a spiral, or by simple contact, for by this means the transmission being made through a smaller space, the intensity of action is proportionally increased.

This experiment, as we have already stated, is due to M. Arago, and it seems at first sight somewhat at variance with our hypothesis; because we have here an appearance of actual attraction between the iron and the wire, whereas we have supposed that there is no attraction between them. A little consideration will, however, show, that instead of contradicting, this fact will serve to confirm the hypothesis in question.

Let us, for example, conceive W (fig. 8) to denote the section of our conducting wire descending vertically from the zinc end of the battery; then, the first and direct action of this wire will be to excite magnetism in any small particle of iron *n s*, according to the direction indicated by the letters in the figure, and agreeably to what has been stated in Experiment I.

After which, the action of the wire will be to urge the point *n* in the line *n n′*, perpendicular to *n* W, and the point *s*, in the line *s s′*, perpendicular to *s* W; and, in consequence of the combined

action of these forces, the particle *n s* ought necessarily to approach the wire in the same way as it would do by a direct attractive force. This effect is therefore still consistent with our hypothesis, and strongly confirmatory of it.

We have seen that by giving the conducting wire a spiral form, its power of magnetism is much increased; and in the same way the power of the wire on the iron filings may be rendered very great. The best form for the spiral, however, here, is that in which the wire lies all in one plane, as in (fig. 24). This being connected by its two extremities with the poles of the battery, will take up an astonishing quantity of filings, which, by their reciprocal attraction towards each other, exhibit the most pleasing appearance.

Experiment VI.

To exhibit the rotation of a magnet round a galvanic wire.

271. Let A B E D (fig. 9) represent a cup of glass, wood, or any other non-conductor, and N S a small magnet, having a hole drilled at S, whereby it may be fixed by a short piece of silk S *c′*, to the copper wire *c′* C, passing through the foot of the cup;* and let mercury be poured into the latter

* The small metal cup at C is soldered to the wire, and having a little quicksilver in it, furnishes the best means of

till the needle floats nearly vertical. Conceive, also, Z z′ to be part of the conducting wire, descending from the zinc side of the battery, and slightly immersed in the quicksilver. If now the contact be made at C with the copper side of the battery, the magnet N S begins to rotate about the wire Z z′, passing towards the left hand of the observer, situated according to the principles of (art. 258). This rotation will be greater or less according to the power of the battery, and will continue while there is sufficient force in the latter to overcome the resistance of the quicksilver to the motion of the magnet. If the descending wire proceed from the copper side of the battery, the motion will take place in a contrary direction, that is, from left to right.

Or, if the contact remain the same, and the magnet inverted, then also the motion will be reversed; but if the contact and magnet be both reversed, the rotation will be the same as in the first instance.

This highly curious and important experiment, which is due to Mr. Faraday, of the Royal Institution, is immediately explained by our hypothesis;

making the contact, the ends of the wire being amalgamated for that purpose. It is not, however, actually necessary to employ this mode, as the simple contact of the wires is sufficient; but I have always found the connection shown in the figure to succeed best.

according to which, the extremity N of the magnet is always acted upon by two forces, one the galvanic force, which is tangential to the wire, and the other the tension of the silk S c', in the direction of the needle. Let this latter be resolved into two forces, one vertical and the other horizontal, and we shall find the extremity N under the influence of two horizontal forces, one always central and the other tangential. The result of which must be a rotation of that point about the wire; and it will be made with the position and arrangement shown in the figure, from right to left, the observer supposing himself situated as in (art. 258).

Experiment VII.

To exhibit the rotation of a galvanic wire about the magnet.

272. Let A B D E (fig. 10) be a cup or vessel of wood or glass, and N S a magnet passing tight through its foot; Z z a conducting wire descending from the zinc side of the battery, and rendered free to move by the chain connection at g. Let mercury be poured into the vessel till the extremity of the wire is slightly immersed in it. Then the contact being made at C (which by means of the wire D C, communicates with the quicksilver) the wire g z will immediately assume a rapid rotatory motion, much greater than in the former

case, the resistance being very considerably diminished by the mode of suspension. The direction of the motion, according to the arrangement in the figure, being from left to right, to a person coinciding in position with the magnet. It may, however, be reversed by reversing the magnet, or by changing the contact, as in the preceding cases.

This experiment is also due to Mr. Faraday, and its explanation is the same as the last; for since when the magnet is free it will, as we have seen, revolve about the wire from right to left, it follows that, when the magnet is fixed and the wire free, the latter will revolve in an opposite direction, (the action and re-action between the wire and the magnet being reciprocal) which is still however towards the left of a person supposed now as coinciding in position with the magnet, and his head to the north.

The same otherwise.

273. The resistance being very inconsiderable in this experiment, it may be exhibited in a more simple manner. For instance, instead of piercing the foot of the cup, as in the figure referred to, it will be sufficient to use a tea-saucer, or any other shallow vessel, and to bring a strong magnet as near to it as possible under the table, when the motion will take place precisely in the same manner as above.

By this means also we may establish a most

important fact; viz. that it is indifferent, as to the result of the experiment, what may be the position of the magnet; that is to say, if we keep the extremity of it as nearly as possible under the centre of the vessel, we may hold it either vertical or horizontal, or incline it in any angle, and at any azimuth, without greatly changing the rate of the rotation; it being always understood that the magnet should be of considerable length, in order that its other pole may not affect the motion of the wire. This result ought necessarily to be obtained, for in explaining the cause of the motion of the magnet about the wire, in Experiment V, we have made no reference to the position of the magnetic particles themselves; the motion, according to the principles we have adopted, would take place exactly the same (except as far as regards the mechanical difficulty) if the magnet could have been placed horizontally instead of vertically, and therefore the rotation of the wire about the magnet ought to be the same in both cases; viz. with the magnet placed either vertically or horizontally, and consequently also at all intermediate angles of inclination.

Experiment VIII.

Exhibiting the two preceding rotations by Mr. Faraday's *apparatus.*

274. The machine for the exhibition of these

motions, according to Mr. Faraday's construction, is shown in (fig. 12). A B C D is a stand of wood, E F a brass pillar, F G a fore arm or projecting piece of brass, through the extremity of which passes the wire L H K; at L, there is a sort of ball and socket joint; the socket being in the upper part, and the ball fitting it, on the small wire L *m*. Both the socket and ball are amalgamated, and a piece of silk fixed to the ball, or head of the wire, passes through a hole drilled in the wire L H, and by which the smaller wire is suspended, thereby preserving the contact, and leaving to the latter a perfect freedom of motion: *a b* is a glass cup having a hole through its foot, into which is inserted a copper tube, soldered to a copper disc just the size of the bottom of the glass, and which disc is cemented to the foot of the latter.

The wire Z *z* is also soldered to another copper disc, upon which the glass rests; and by which the contact is carried on from Z to the quicksilver in the cup, and thence to the wire *m* L; lastly, a small magnet *n s* is inserted into the copper tube, passing through the stem of the glass above mentioned.

The foot of the cup *c d* is pierced, and discs of copper applied as in the cup *a b*; but the wire passing through the foot is solid, and to it is fixed, by a short string, the small magnet *n s*, which is thus free to revolve about the descending wire

H K; quicksilver, as in the preceding cases, being poured into the cup, till the wire H K is slightly immersed in it at K. The contact with the battery being now made at Z and C′, the motions will take place as described in the two last experiments; viz. the magnet *n s* in the one cup will revolve about the wire K, while the wire L *m* will at the same time be revolving about the other magnet *n s*.

If the cup *c d* be placed where the cup *a b* is represented, then the magnet and wire being both free, they will revolve about each other, and thus produce a pleasing variety in the experiment.

A section of this machine is shown in (fig. 13).

275. Mr. Faraday also describes another apparatus, which requires a less galvanic action than the former to produce the rotation. This is shown in (fig. 14); it consists of a piece of glass tube, the bottom part of which is closed by a cork, and through it is passed a small piece of soft iron wire, so as to project above and below the cork. A little mercury is then poured in, to form a channel between the iron wire and the glass tube. The upper orifice is also closed by a cork, through which a piece of platinum wire passes, being terminated within by a loop; another piece of wire hangs from this by a loop, and its lower end, which dips a very little way into the mercury, being amalgamated, it is preserved from adhering either

to the iron wire or the glass. Things being thus arranged, a very minute galvanic power being applied by a contact with the lower and upper end of the apparatus, and the pole of a strong magnet being applied to the external end of the lower iron wire, the moveable wire within begins rapidly to rotate round the temporary magnet thus formed; and which rotation may be inverted either by changing the contact or by inverting the magnet. Mr. Faraday states that this instrument is so sensible that a rotation has been produced in it by two plates, each only one inch square.

Experiment IX.

To exhibit the rotation of a magnet on its axis by the effect of a galvanic wire.

276. Let A B D E (fig. 11. plate 5) represent a cup of glass or wood, N S a magnet, having at its lower extremity a fine steel point, inserted in the agate *a*; *b c* is a thin slip of brass or ivory, having a hole through which the magnet passes freely, and by means of which it is kept perpendicular: at the upper extremity N of the magnet, is a thin cylinder, as a piece of quill, forming a cup or reservoir *z* to receive a small quantity of quicksilver; and into this is inserted the wire Z, amalgamated at its lowest point, and C *c* is a stout wire passing through the side of the cup into the

quicksilver. Then, the contact being made at C and Z, the magnet will begin to revolve on its axis, with a very astonishing velocity, and continue in motion while the power of the battery lasts.

This pleasing experiment is due to M. Ampere, who employs only a piece of platinum attached to the magnet, to produce, by its superior gravity, a vertical position of the latter in the mercury; the upper wire being then inserted into the quicksilver in the cylinder z, and the other wire into the cup C, the motion is produced exactly as above described: the greatest freedom of motion is, however, given by the apparatus shown in the figure. The explanation of this rotation is very obvious according to the hypothesis we have adopted, for the tangential force of the wire acting upon the magnetic particles on the surface of the magnet, must necessarily produce the rotation in question, on precisely the same principles as the magnet is made to revolve about the wire in the fifth experiment.

Experiment X.

To exhibit the rotation of a galvanic wire on its axis by the action of a magnet.

277. Let N S (fig. 15) be a magnet, represented as broken in the figure, but which is fixed, in the experiment, in a foot, in order to keep it vertical, and

let *a b c d* be a light hollow copper or brass cylinder having a steel point passing downwards into the agate cup *f*, fixed to the upper end of the magnet; and let *e* be a small tube or quill fixed on the wire passing through the top of the cylinder, holding a little quicksilver, and receiving into it the descending conducting wire Z. A B is a piece of wood turned to fit on the cylindrical magnet N S, which has a hollow groove on its upper surface to receive a quantity of quicksilver, into which the lower edge of the cylinder *a d* is slightly immersed, the surface being covered with weak dilute nitric acid. A C is a wire passing into the quicksilver. It is obvious that thus (the contact being made at Z and C) the galvanic circuit is carried from Z through the cylinder *a b c d*, thence to the quicksilver, and hence again through the wire A C to the other extremity of the battery, whereby the cylinder *a b c d* is made to become a part of the conducting wire, and it will be found to revolve on its axis with a great velocity, fully equal to that of the magnet in the last experiment; the direction of the motion, with the arrangement shown in the figure, being from left to right, to a person coinciding in position with the magnet. If we conceive the cylinder to consist of an infinite number of wires, the explanation of this motion is the same as in Experiment VII.

Experiment XI.

To exhibit a quicksilver vortex by means of a galvanic wire and magnet.

278. To perform this experiment it is only necessary to take any shallow non-conducting vessel and put into it a quantity of pure mercury, into which is to be inserted the conducting wires Z, C, proceeding respectively from the zinc and copper sides of the battery. And if now the north end of a strong magnet be brought under the vessel, the quicksilver round the wire C will begin to revolve about the same, forming a beautiful vortex, the direction of the motion being from left to right. If the magnet be removed under the other wire the same kind of motion will be produced, but its direction will be reversed, and the same change of motion will take place, of course, in each case, by changing the end of the magnet.

The explanation here is precisely the same as in the last experiment; the moveable part of the conductor in this case, owing its mobility to its fluid nature, whereas in the former it is due to the peculiar mode of suspension.

This very elegant experiment was first made by Sir H. Davy, and although it is referred to in this place for the sake of arrangement and concatenation, it was made prior to the two former, the

last of which I was led to institute from the hints furnished by this.

Experiment XII.

To exhibit the rotation of the galvanic wire independently of the galvanic battery.

279. For this purpose we must employ the apparatus exhibited in (fig. 16) where A B C D is a small copper vessel about $2\frac{1}{2}$ inches high, and the same in diameter; *a b c d* is another small cylinder of copper, of the same height, soldered to the former vessel at its lower end *d c*, a hole being left in the bottom of the former to receive it. The cylinder *a b c d* is therefore open, and will admit a cylindrical magnet to be passed up, and it will at the same time hold a quantity of dilute acid within the space A D *d a b c* B C: *z z′* is a zinc cylinder, very light, of rather less altitude than the copper one. To the cylinders *a b* and *z z′* are soldered two copper wires, as shown in the figure, the upper one having a steel point proceeding from E downwards and resting in a small metal hole at F, and consequently the cylinder *z z′* will be free to move upon its point of suspension at F.

Things being thus prepared, and the acid placed in the cell as above described, insert through the interior cylinder the north end of a strong cylindrical magnet, and balance the whole apparatus

upon it; when immediately the zinc cylinder will begin to revolve, with a greater or less velocity, according to the strength of the acid, the freedom of motion, and the power of the magnet. I have frequently with this simple apparatus produced a motion amounting to 120 rotations per minute. The only difference between this and the other rotations we have described is, that the galvanic power is here produced by the apparatus itself, instead of having recourse to the battery.

For it is obvious that the wire from z z′ to E, may be considered as a conductor proceeding from the zinc, and the wire from *a b* to F, as one from the copper side of the battery; and consequently, the same effect is to be expected here as in the preceding cases. It is unnecessary to add, that with the north end of the magnet upwards, the motion is from left to right, and the contrary with the magnet reversed.

This experiment is due to M. Ampere.

The same otherwise.

280. A very pleasing addition has been made to this apparatus by Mr. J. Marsh. It consists in having a second point descending from F, which is made to rest in an agate cup, fixed on the top of the magnet, (fig. 17) and upon which the whole machine is balanced, having a perfect freedom of motion; and to preserve this balance, the magnet

is placed vertically in a foot. The machine being now charged with acid, a compound motion takes place, the zinc cylinder revolving in one direction and the copper vessel in another, producing thus a very pleasing effect; the latter however is by no means so rapid as the other, in consequence of the weight of the acid, and in fact that of the whole machine being supported on the lower point.

This young man, to whose ingenuity and industry I am much indebted for the success of my experiments, is at present employed in an inferior situation in the laboratory of the royal arsenal; but his dexterity as a workman, his practical chemical knowledge, and his regular conduct, are qualifications which render him deserving of a more respectable and profitable occupation.

Experiment XIII.

To show the effect of a horse-shoe magnet on a freely suspended galvanic wire.

281. Let Z z (fig. 18) denote a part of the galvanic wire, freely suspended by the chain connection at *o*, proceeding from the zinc end of a battery, its lower extremity being amalgamated and slightly immersed in a reservoir of pure mercury, having a connection at C with the other extremity of the battery. N S is a horse-shoe magnet, posited as shown in the figure.

The contact being now made at C and Z, the

hanging part of the wire $o\ z$ will be thrown out of the mercury into the position $o\ z'$; the contact being thus broken, it falls by its own gravity into the mercury, by which means the contact being renewed it is again projected, and so on with an extraordinary rapidity; and if the position of the magnet be reversed, or the contact be changed, the direction of the motion will be changed also, but the effect will be the same.

This singular motion may be still explained by the hypothesis that has been advanced; for the wire having a tendency to pass round the north end of the magnet to the right hand, and round the south end to the left hand, is urged by equal forces directly in a line with the open space of the magnet, the equality of the two forces preventing the rotatory motion about either, but both conspiring to give to the wire the rectilineal motion which has been described.

This experiment is also due to Mr. J. Marsh.

Experiment XIV.

To exhibit a wheel and axle rotation by means of a horse-shoe magnet.

282. The machine by which this motion is produced is represented in (fig. 19), where A B is a rectangular piece of hard wood, C D an upright wooden pillar, D E a piece of stout brass or

copper wire, and *a b* a somewhat smaller wire, soldered upon it at E, on the lower side of which the wheel W, of thin copper, turns freely; *h f* is a small reservoir for mercury, sunk in the wood, and *g i* a narrow channel running into it: H H is a strong horse-shoe magnet. Mercury being now poured into the reservoir *f g*, till the tips of the wheel are slightly immersed in it, and the surface covered with weak dilute nitric acid, let the connection with the battery be made at *i* and D, and the wheel W will immediately begin to rotate with a great velocity. If the contact be changed, or if the magnet be inverted, the motion of the wheel will be reversed; but in general, the best effect is produced when the wheel revolves inwards. The suspension of the wheel, which I find to answer the best, is shown in (fig. 20). This is a necessary consequence of the motion described in the last experiment, by which it was suggested, and is explained on the same principles.

Experiment XV.

To exhibit a compound wheel and axle rotation with two horse-shoe magnets.

283. The machine for producing this motion is shown in (fig. 21); A B G D is a rectangular piece of board, having two grooves, about half an inch deep, cut in it parallel to its length. C *p*, Z *q*, are two wires having cups for connection

at Z and C, and each passing into its respective groove *a b*, *c d*, filled with mercury; into which are slightly immersed the points of the wheels W, W′; these being fixed on an axle W W′, and resting upon the two supports *m n*, *r s*, brought to a fine edge at *n* and *s*, in order to reduce the friction as much as possible, and to give the greater freedom of motion. N S are two horse-shoe magnets, posited as in the figure, with the like poles interior and exterior of the wheels.

The apparatus being thus prepared, and the contact made at Z and C, the wheels will begin to rotate, and in a very short time will acquire a velocity exceeding very considerably any of the motions hitherto described.

It is unnecessary to say that by changing the contact, or by inverting the magnets, the direction of the rotation will be also changed. The usual precaution of covering the surface of the mercury with weak dilute nitric acid, will increase the rapidity of rotation, but it is not actually necessary in this case.

Experiment XVI.

To exhibit the terrestrial directive quality of a galvanic wire.

284. It was not long after the first experiments of Mr. Œrsted, that the question naturally suggested itself, "Has the galvanic wire a directive,

as well as a general, magnetic power?" This question was soon answered in the affirmative by M. Ampere, who made use of the following ingenious construction:

A B (fig. 22) represents a piece of wood fixed to any convenient support, through which pass the two wires G, E, and where they remain fixed. At their upper and lower extremities are soldered the small metal cups *a*, *b*, *c*, *d*. D H I K, &c. is a part of the conducting wire, bent into the form shown in the figure, having small steel points soldered upon it at *c* and *d*. These points are inserted into the cups *c*, *d*, the upper one only resting on the base of its cup, the other being merely brought into contact with *d*, by a little quicksilver placed in it for that purpose, by which means the rectangle has a great freedom of motion given to it, the only solid contact being on the point *c*. Mercury is also poured into the other cups, for the sake of a more perfect and certain communication than that afforded by the mere juxtaposition of the wires.

The apparatus being thus prepared, the two wires proceeding from the copper and zinc sides of the battery are inserted into the cups *a*, *b*, and thus the connection is established; first by means of the wire G with the cup *c*, thence by means of the contact of the point with the cup and mercury, it is carried forward from *c* through the

rectangle, to the cup *d*, whence it proceeds to the cup *a*.

We have already seen that of this connecting wire, the part from *c* to *d* has a perfect freedom of motion upon the point at *c*, and will therefore obey any exciting force. This force, in the experiment in question, is the magnetic influence of the earth, and in consequence of which the rectangle, immediately the contact is made, places its plane perpendicularly to the plane of the magnetic meridian, and to which position it will always return after a few vibrations, if it be drawn out of it by the hand, or otherwise.

This arrangement of the moveable conductor is perfectly consistent with our hypothesis, as is obvious without any farther illustration than what has been given in several preceding experiments.

285. A differently formed wire, and a more simple mode of suspension, is shown in (fig. 24.) Here a brass or copper wire A C, rests at its bent end A, in a cup containing a little mercury, and is very moveable in azimuth round this point. The other end passes through the centre of a circular piece of pasteboard, and then forms spiral turnings in the plane of this circular piece. The wire is attached by thread or silk to the pasteboard disc, and at the point B it turns and descends till its extremity reaches the quicksilver in the cup D.

The communication being now made at A and D with the battery, the spiral will immediately arrange itself, as in the last case, in a plane perpendicular to the magnetic meridian. This experiment is originally due to M. Ampere, but the mode of suspension described is that of Professor Van den Boss. *See Edin. Journ. of Science*, No. XII.

A needle upon a different construction, also due to M. Ampere, is shown in (fig. 23.)

The same otherwise.

286. The directive quality of the galvanic wire has been since exhibited in a variety of ways, much more simple than that above described, of which we shall only state the following:

M. de la Rive's apparatus.—This consists of a small galvanic combination attached to a cork; the plate of zinc is nearly half an inch wide, and extends about one and a half or two inches below its cork, its upper end passing through the same; the slip of copper is of equal width to the zinc, but passes round it, being thus opposed to both its surfaces, as in Dr. Wollaston's construction; its upper end also appears through the cork. A piece of copper wire, covered with silk thread, is coiled five or six times, and tied together so as to form a ring about an inch in diameter, and the ends of the wire are connected, by solder, one with

the zinc, and the other with the copper slip above the cork. See (fig. 25).

When this small apparatus is placed in water, slightly acidulated with sulphuric or nitric acid, the ring becomes highly magnetic, and will arrange itself in a plane perpendicular to the magnetic meridian, or it will at least indicate a tendency to take up that position, but the escape of the bubbles, arising from the decomposition of the water, prevents it from preserving a fixed direction.

Its magnetic qualities, however, are more obviously shown by bringing to it a strong magnet. The one I made use of is cylindrical, about three quarters in diameter, and 18 inches in length. This being applied at the distance of several inches, the ring was immediately attracted, or repelled, accordingly as one or the other of the poles of the magnet was presented, or accordingly as one or the other side of the wire was opposed to the latter. When the result of the application is attraction, the cork will advance towards the extremity of the magnet, and if the latter be held horizontally, and in a line with the centre of the former, this will continue to advance till the pole of the magnet is within the ring, and then proceed with considerable velocity till it reaches the middle of the magnet, where it remains perfectly stationary. If now the magnet be withdrawn, and changed end for end,

and re-introduced into the ring, the latter will go off from the magnet, turn itself round when quite free from it, again advance, and settle itself as before in the centre.

This very simple apparatus, which may be made at the expense of about a shilling, throws great light upon the nature of the electro magnetic action, and proves most satisfactorily that, notwithstanding the intimate relation between the electro magnetic and simple magnetic fluids, they are not identical; for no possible arrangement of simple magnets can be made that would lead one of them beyond the pole of another to find its state of equilibrium in the middle of the latter. At the same time all the above facts will be found perfectly consistent with the hypothesis that has been advanced; for it will be seen, when the wire and cork are in equilibrio, as above stated, that an observer, conceiving himself situated as in (art. 258), will have the north end of the magnet to his left hand, and the south to his right, at equal distances, and acting therefore with equal and opposite powers; consequently the wire itself ought to be in equilibrio, and when disturbed from it will have a tendency to regain it, and hence be subject to all the conditions of motions that have been described. This is in fact very similar to experiment 4, the difference only consisting in this, that in the present case the wire is moveable and the

magnet fixed, whereas in the former the wire was fixed and the magnet free; the explanation is of course the same in both.

Another form of this apparatus is shown in (fig. 26.)

Both the above apparatuses are much improved by fixing to the cork a light glass cylinder A B to contain the acid, instead of floating them in it; the apparatus may then be floated on common water, and all the facts exhibited as above described.

This appendage to the original construction is due to Mr. James Marsh,* already mentioned.

287. *Apparatus of Prof. Van den Boss.*—Here C D (fig. 27) is a copper plate, E G a similar one of zinc, about an inch square, kept from touching each other by the interposition of some small piece of wood: both plates are attached and suspended to slender brass wires P and R. The wire P enters at P, in the hollow space formed by a case of very thin quills inserted into each other, about 6 or 7 inches long. The end of the wire comes out of the quill at the extremity T, and

* This ingenious workman has just completed a portable electro galvanic apparatus; which within the space of little more than a cubic foot, contains not only the necessary galvanic combination, but also all the instruments necessary for repeating nearly the whole of the experiments detailed in this Section.

returns, being wound as a spiral about it to the other extremity V, where it again enters the quill, and proceeds in a right line to R, where coming out it descends, and is attached to the other plate. The whole is suspended in equilibrio to a piece of untwisted silk X. The plates are now dipped into dilute acid, and the whole is suspended at X, when immediately the magnetic quality of the wire becomes manifest; but, like the former instrument, it is not so sensible to the terrestrial as to the action of a strong artificial magnet, with which its extremities T and V may be attracted or repelled, according as the one or the other pole of the magnet is applied; and which ought necessarily to be the case agreeably to the explanation given in the preceding case.

Experiment XVII.

To examine the inclination of a freely suspended galvanic wire as affected by the terrestrial magnetism.

288. This is an experiment of M. Ampere, in which he employs the apparatus exhibited in (fig. 28), where the galvanic circuit is carried on from the extremity of the battery towards V, passes by V S, through the steel pivot *k*, placed on the metallic plate N, and thence through the rectangle A B C D; whence, passing through the tube *x y*,

which serves as an axis for the machine, it is carried by means of a second steel pivot to the other extremity of the battery towards R. The moment the connection was made, M. Ampere found the moveable part of this conductor in a state of vibration, which after a short time subsided; when the plane of the rectangle was found to coincide with what has been denominated the magnetic equator, or plane of no attraction; that is, with a plane perpendicular to the direction of the dipping needle.

It will of course be understood, that the axis of the machine must in the first instance be placed very exactly at right angles to the magnetic meridian, that the whole requires to be very nicely balanced, and that a little mercury be placed on the plates M N to render the contact the more perfect.

The lozenge *z w* is made of very light wood, and being fixed on the axis, serves to keep the rectangle in its proper form.

That the machine ought, according to our hypothesis, to assume this direction is obvious from all that has been previously stated, and therefore requires no particular illustration.

Experiment XVIII.

To exhibit the action of the terrestrial magnetism upon a galvanic wire freely suspended.

289. Let A B G D (fig. 29) represent a rectan-

gular piece of hard wood, having two grooves *a b*, *c d*, cut in it, parallel to its length, about half an inch in depth, which are to be filled with quicksilver for the experiment. C *p*, Z *q* are wires fixed in the board and passing each into its respective groove, with cups for making the connection with the battery at Z and C. O *m* is a long piece of silk proceeding from the ceiling, or some other convenient place, and to which is tied the wire *k m n*, bent as in the figure, the points *k* and *n* being slightly immersed in the quicksilver. If now the connection be made at Z and C with the zinc and copper sides of the battery, the moveable part *k m n* of the galvanic circuit, which has a great freedom of motion, will be projected towards the extremity A B of the board, and if the contact be changed, by making the zinc connection at C and the copper at Z, the wire will be driven towards the other extremity. As no magnet is introduced in this experiment, we have a right to attribute the motion to the effect of the terrestrial magnetism, the direction of it corresponding precisely with what we ought to expect from such action. For the terrestrial magnetism of our latitude being of the same kind as that exhibited by the southern pole of a magnet, the moveable wire ought to pass from right to left in the first case, and from left to right in the second, to an observer situated as described in (art. 258) ; viz. as forming a part of the galvanic

circuit, and with his head towards the zinc end of the battery; that is to say, with the first contact the wire ought to be projected towards A B, and with the second towards D G.

To prove that the motion proceeds from this cause, let the south pole of a strong magnet be brought under the board between Z and C, and make the contact again; and the same motion will take place, but in a much stronger degree, the wire being now thrown very forcibly out of the mercury.

The effect therefore being precisely of the same character, but much more powerful in the latter case than in the former, we have a right to conclude that the cause of the motion in both cases is of a like nature, the one proceeding from a southern polarity artificially produced, and the other from the natural magnetic action of the terrestrial sphere, as stated by Mr. Faraday, to whom we are indebted for this interesting experiment.

Experiment XIX.

To produce a rotation of the galvanic wire by means of the terrestrial magnetism.

290. This is also an experiment due to Mr. Faraday, and which proves, in the most satisfactory manner, the influence of the terrestrial magnetism in the production of a rotatory motion. It is per-

formed as follows: a very light copper, or platina wire, about six inches long, is suspended very freely from a larger wire proceeding from either end of the battery, by means of the chain connection described in several of the preceding experiments, and at its lower extremity a small piece of cork is attached in order to keep the wire buoyant on a basin of pure mercury, about 10 inches in diameter. The wire by which the above small moveable piece is suspended, is then so much depressed that the proposed revolving wire slopes at an angle of about 40° with the horizon; in this state the circuit of the battery is completed through the mercury in the basin and the other conducting wire, when immediately the short wire commences a rotation, as it would do about the south end of a magnet, but in a proportionally less degree, as the directive power of the earth is less than that of a magnet of the kind here supposed.

This similarity of action naturally leads us to infer a similar cause, and that this cause is no other than the terrestrial magnetism; still, however, in order to render this conclusion the more indisputable, Mr. Faraday changed the inclination of the wire, making it first equal to the angle of the dip; and when under these circumstances the wire was placed so as to coincide with the dip itself; viz. when placed in the magnetic meridian,

sloping from south to north, there was no motion; and when the angle was still farther increased so as to exceed the angle of the dip, it was projected in two different directions according as it was made to slope to the north or to the south, which is precisely what ought to be the case on the supposition of the motion being caused by the magnetism of the earth.

For let $o\,z$, $o\,z'$, in (fig. 30 and 31), represent the freely suspended wire in the plane of the meridian, sloping respectively to the north and south: and let N S in both figures denote the direction of the terrestrial magnetism, then it is obvious in the first of these figures, that whether the slope be towards the north or towards the south, it will be always on the same side of the line N S, and will in both cases be projected in the same direction, with respect to the observer, situated as supposed in (art. 258), and consequently in opposite directions as referred to the circular rotation of the extremity z or z'. But when the slope is less than the dip, then the wire in its two positions being found on opposite sides of the line of direction, and passing still to the same hand of an observer situated in the wire, a rotation will ensue similar to those that have been described in our experiments 7 and 8.

Experiment XX.

To exhibit the action of two galvanic wires on each other.

291. The apparatus which I employed for this purpose is shown in (fig. 32), where A B represents a rectangular board, and D, E, two upright pieces of wood, carrying each a cross piece at top with several holes for receiving the cups *m*, *m′*, *n n′* which by this means may be placed at different distances; a little mercury is poured into each of these so as to communicate with the wires inserted through the side of the cup, and terminate with fine points. The wires *w a a′ w′*, *w b b′ w′* are bent as shown in the figure, and have small holes drilled at *a*, *a′*, *b*, *b′*, whereby they may be hung freely upon the points of the wires *m*, *m′*, &c. and carrying small weights *w w′*, &c. in order to bring the points of suspension to correspond as nearly as possible with the centre of gravity, whereby the wires are moved by the least force. The conducting wires from the extremities of the battery Z and C are terminated as represented in the figure, and being each brought to the respective cups, so that *z z′* are respectively inserted in the cups *m n*, and *c′ c* into the cups *m′ n′*, the circuit will be made through the two wires *a a′*, *b b′* in the same direction, and these being free to move about the points in the respective cups, will

be strongly attracted towards each other, even at the distance of several inches.

Let now the branch z of the conducting wire $Z\ z$ be lengthened so that it may pass round the board and be inserted in the cup n', while z' is inserted in the cup m as before; lengthen also the branch c of the conducting wire $C\ c$, passing it round the board and dipping it into the cup n, while c' is immersed in m' as at first; by this means the circuit passes from z to c along the wire $b\ b'$, and from z' to c' along the wire $a\ a'$; in short, the circuits in the two wires are now made in opposite directions, and the wires experience and exhibit a mutual repulsion. Hence we learn, that two galvanic wires, parallel to each other, and in which the circuit is made in the same direction, are attracted towards each other; but they are mutually repelled when the circuit passes in opposite directions, a result first deduced by M. Ampere, and which he has made the foundation of his theory of electro magnetism, by assuming that the powers exhibited by artificial and natural magnets are due to currents of the galvanic fluids circulating in planes perpendicular to their axes; and, that those currents, when parallel to each other and passing in the same direction, are attracted, and when in opposite directions, repelled.

292. Whether this hypothesis and that which I have advanced be under different forms only one and

the same, and if not, which may be considered as the most conclusive and satisfactory are not for me to determine: they are now both in the hands of philosophers, who will judge of them impartially, and adopt that which seems to answer best to the various facts and phenomena that have been, and that may still be, elicited by the ingenious experimenters at present engaged in this interesting inquiry: I must say that I cannot, on M. Ampere's doctrine, satisfactorily explain several of the phenomena exhibited in the preceding experiments; and the following is another case which seems to be at variance with the theory in question; viz.—

293. Let only one of the bent wires, shown in the figure last referred to, be employed, and let it be made a part of the galvanic circuit. If now a long magnet be placed horizontally, with one pole a little below the horizontal part of the wire, and perpendicular to the same, the wire will be strongly attracted, or repelled, according to the pole that is presented. Let us suppose that the wire is attracted; this may be explained by the assumed attraction of the current in the wire, and the parallel currents in the same direction in the magnet, agreeably to M. Ampere's theory; and if it be repelled, the explanation will still subsist by supposing the parallel currents in opposite directions; but if, now, instead of keeping the magnet perpendicular to the direction of the wire, we place it

parallel to it, keeping the same extremity still under the wire, the very same effect is produced; although in this case the supposed magnetic currents, *if before parallel* to that in the wire, are now necessarily *perpendicular to it:* and if, again, the magnet be held vertically, keeping the extremity presented to the wire in its situation, or as nearly so as possible, the same attraction still takes place; and this, whether the extremity in question be above or below; in short, while the pole of the magnet presented to the wire is kept in its position, whatever direction be given to the magnet itself, whether in azimuth or inclination, the same motion takes place, which certainly appears to me to be wholly at variance with the doctrine that M. Ampere has endeavoured to establish. And if, instead of using the magnet, we leave the wire to the action of the terrestrial magnetism only, a similar effect, but in a less degree, is produced every time the connection with the battery is established: and it is the same whether the wire be placed at N and S, E and W, or at any azimuth whatever; a fact which seems to be equally at variance with M. Ampere's theory of terrestrial magnetism.

Whether this ingenious author, for whose talents I entertain the highest respect, will be able to reconcile these phenomena with his theory, I am unable to say. If he can, no one will be more ready than myself to admit his doctrine; being

fully aware of the great advantages which philosophy derives from the reduction of a variety of classes of phenomena to one general principle: at the same time we must be careful not to generalize too quickly; nor in our anxiety to avoid the introduction of a force, hitherto unknown in nature, allow ourselves to leave imperfectly explained some of the most interesting facts yet elicited by experimental philosophy.

ADDENDA

TO

SECTION XII. AND XIII.—PART I.

ON THE MAGNETIC EFFECTS OF IRON MASTS ON THE COMPASS.

It being in contemplation to employ hollow iron masts in ships of war in lieu of those at present in the service, I received a letter from Sir Byam Martin, requesting me to state my opinion as to the probable disturbance which such masts might be expected to produce on the compass. My reply was to this effect, that as the centre of attraction of the iron masts, viz. main-mast and fore-mast, would be considerably above the horizontal plane of the needle, and as all the other iron of the vessel was below that plane; it would follow, that while the latter attracted the north end of the needle in any direction, the effect of the former would be to bring the south end towards the same direction; and that, in consequence, it was not improbable the two powers might so nearly balance each other as to destroy the effects of both at those points which are in these latitudes those of greatest attraction; but I stated, at the same time, my

apprehension whether the power of the masts might not so far exceed that of all the other iron, as to produce as great a deviation as the latter, in addition to the counteraction, but in an opposite direction—a circumstance that might be attended with some danger; because there is no doubt that seamen, in one way or other, (from long practice) actually make an allowance for the usual local attraction, although it is in nine cases out of ten attributed to some other cause. If, therefore, this allowance were made as usual, while the effect was in reality reversed, the worst consequences might be apprehended.

I proposed, however, in order not to leave the determination to a mere matter of opinion, to have my model of a 74-gun ship, mentioned at page 88, fitted up with an iron main-mast, fore-mast and bowsprit, to the proper scale, and thus to ascertain from actual experiment the probable amount of the deviation in question. Having done this, I found, as I had expected, that the introduction of the main-mast had raised the centre of attraction from an angle of 50° below the horizontal plane of the compass, to about 20° above it, and that the power was such as to produce a deviation in the needle to nearly the same amount as before, but in an opposite direction.

The only danger therefore to be apprehended by the use of iron masts is, that those allowances

which long habit has introduced, and the necessity for which is attributed to any cause but the right one, will still continue to be made after the cause itself is reversed: should this be the case, some evil consequences might perhaps follow; and the only means that I am ware of to prevent it, is to urge the necessity of accurate observation, and to employ the means of correction explained in this work, or a better if it can be found.

Simplification of the method of correction described in Section xiii.

The above experiments and deductions, relative to the effect of iron masts, led me to reconsider the principles on which the method of correction, illustrated in the above section, depended. In the course of which it occured to me, that as by the introduction of the iron masts I had changed the place of the centre of attraction from an inclination of about 50° below the horizontal plane to nearly 20° above it, it might be possible in all cases to introduce the usual correcting plate, or a larger surface of iron, at a greater distance, in such a situation aft of the compass as to bring the common centre of attraction vertically below the pivot of the needle; in which case every species of deviation would be completely counteracted and destroyed, and the needle be as free and stand as true as if there were no iron on board.

I lost no time in submitting this idea to the test of experiment; and even on the first trial I had every reason to believe that it was correctly founded, and was much gratified on a second trial to find it completely successful; the needle under every direction of the model standing as truly in the meridian as it would have done had there been no iron on board. But still, after the caution I have observed in the several deductions in the preceding parts of this work, I am unwilling to offer a positive opinion on the efficacy of this method, although I should much like to have a trial of it made in a ship at large. One difficulty doubtless is, that, by the means proposed, the centre of attraction will necessarily fall very near the needle, and consequently a little inaccuracy of adjustment, or even the rolling or inclination of the vessel, may so far displace its verticality as to produce irregularities in the results, which are not easily detected under the more favourable and uniform action of the model. The suggestion, however, will not I hope be thought wholly undeserving of attention.

LONDON: PRINTED BY A. APPLEGATH, STAMFORD-STREET.

ADDITIONAL ERRATA.

Page 81, *for* $(\tan x + a)$ *read* $\tan (x + a)$.

200, for *see*, read *sec.*

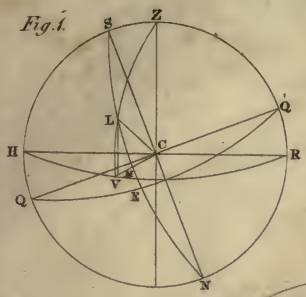
204, two lines were left out; this page and pages 239—242 have been reprinted, and may be had at the Publishers.

211, line 11, from bottom, *for* log 1·65642, *read* $-1{\cdot}65642$.

239, *for* $\frac{1}{d} - \frac{1}{\sqrt{(d^2 + l^2)}}$ *read* $\frac{1}{d}$ arc.tan $\frac{l}{d}$

This occurs only in a few copies; see above, p. 204.

Fig. 1.



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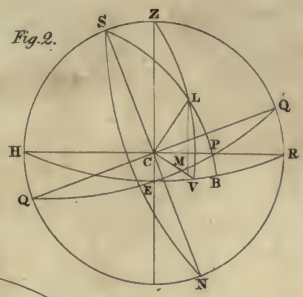

Fig. 3.

Fig. 4.

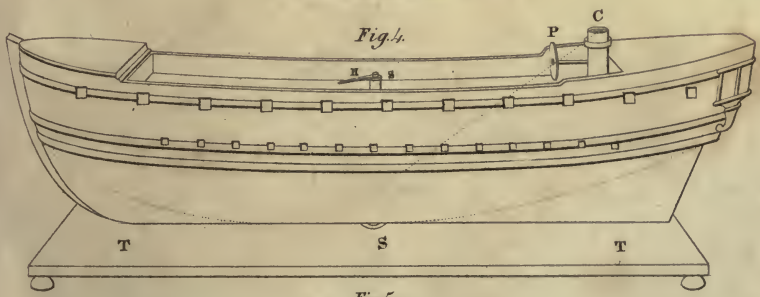

Fig. 5.

Barlow. del. W. Lowry. sculp.

Magnetism.

Plate. 2

Fig. 7.

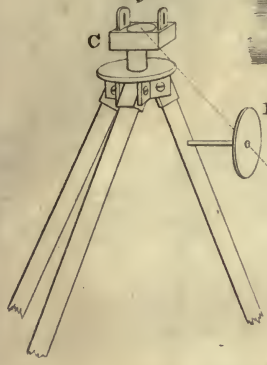

Fig. 6

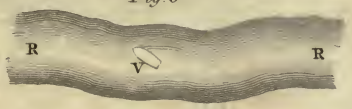

Fig. 8.

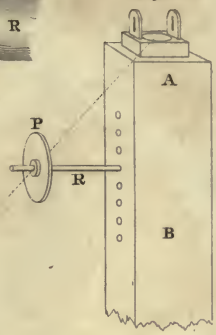

Fig. 9.

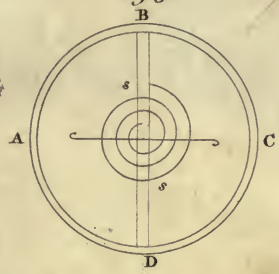

Fig. 10.

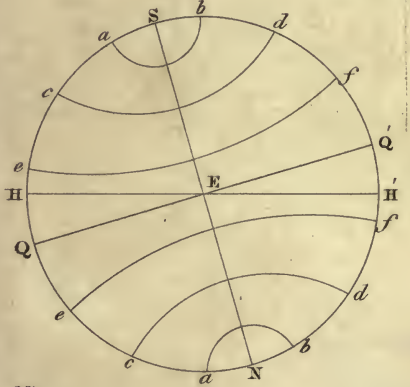

Fig. 14

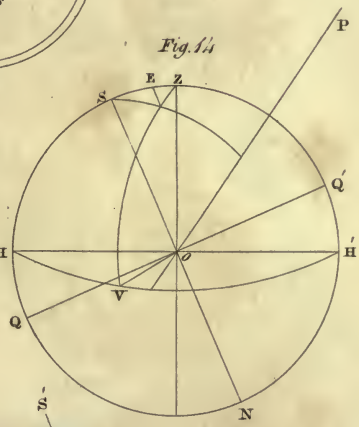

Fig. 11

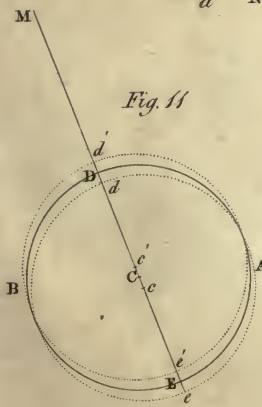

Fig. 12

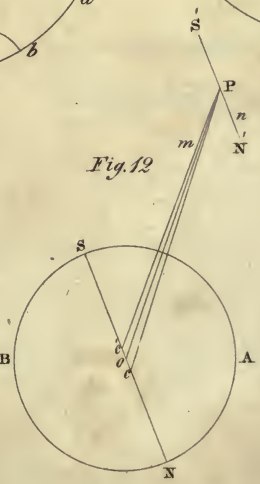

Fig. 13

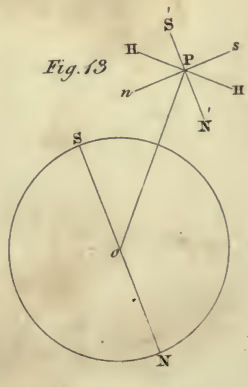

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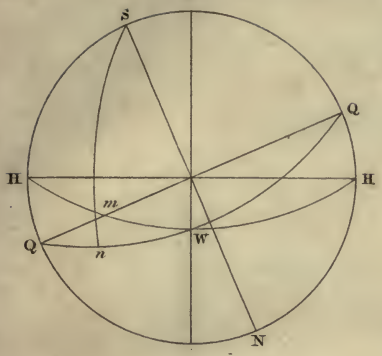
Fig. 15



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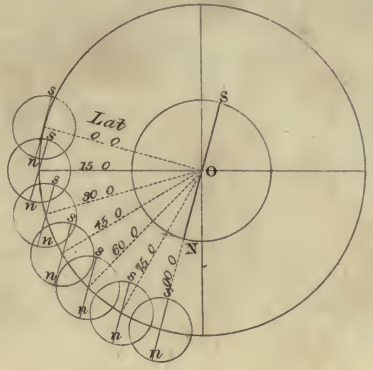

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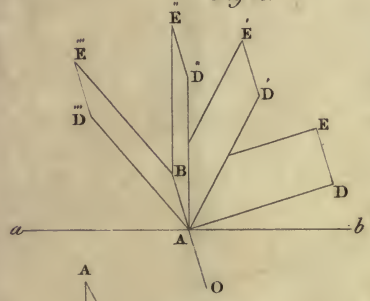

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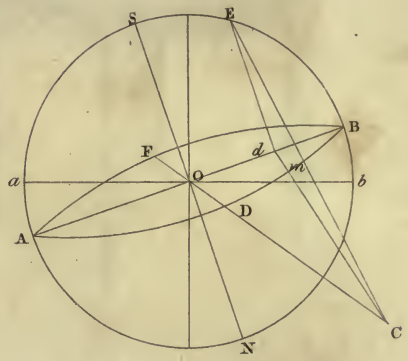

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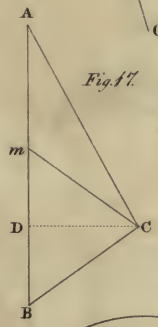

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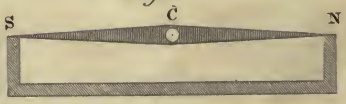

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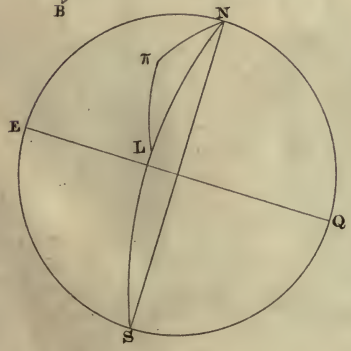

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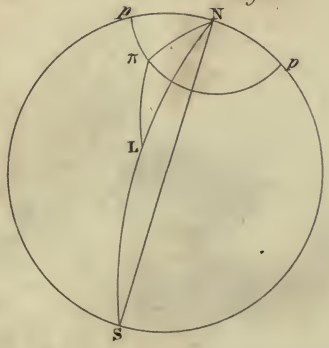

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Fig. 4

Fig. 6

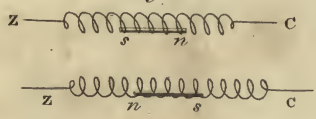


Fig. 2.

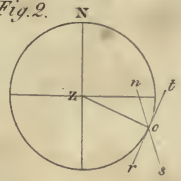


Fig. 3

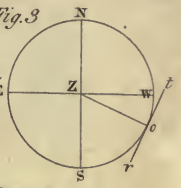


Fig. 1

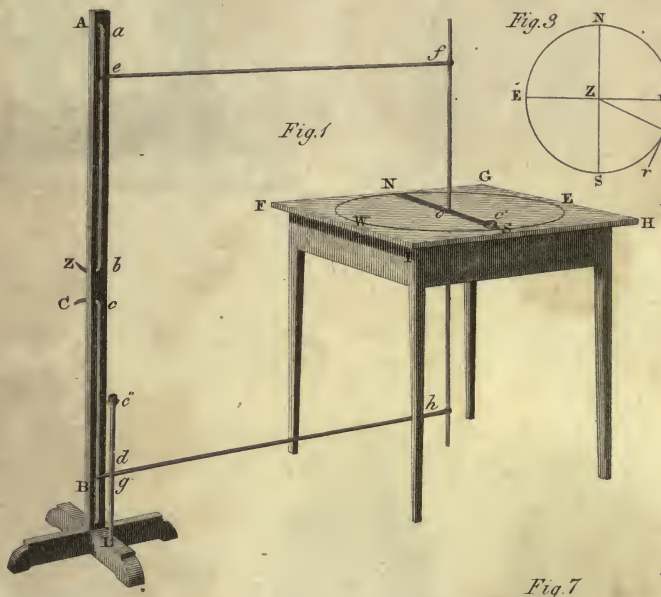


Fig. 7

Fig. 9

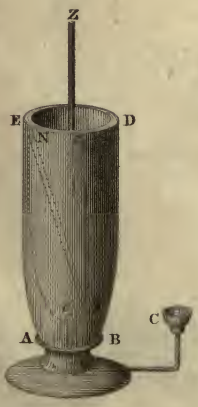
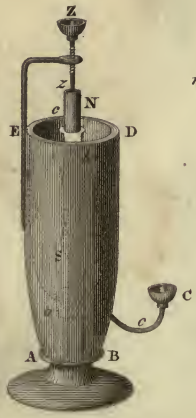


Fig. 11



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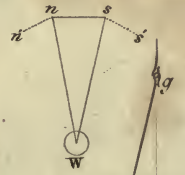

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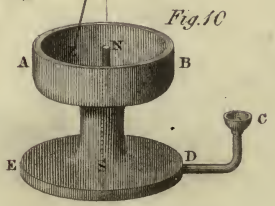

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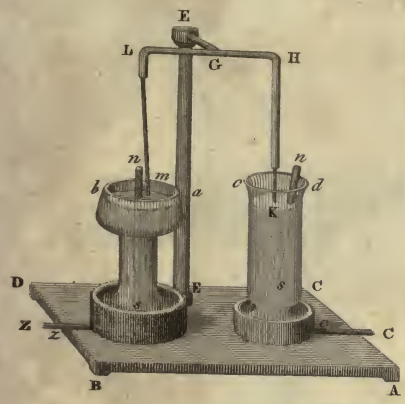

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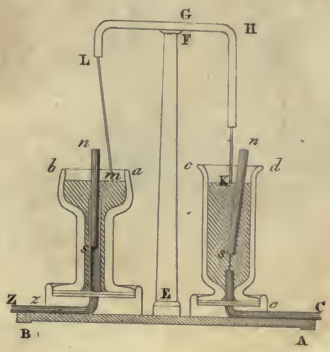

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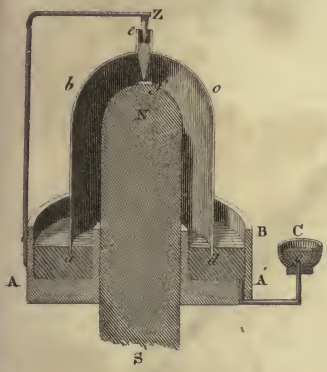

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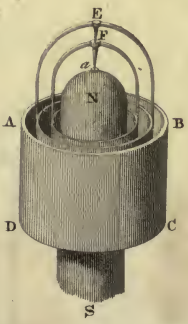

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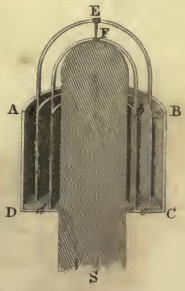

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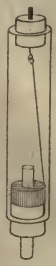

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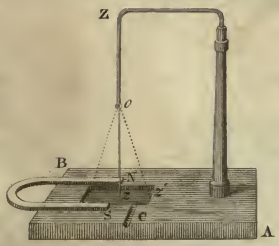

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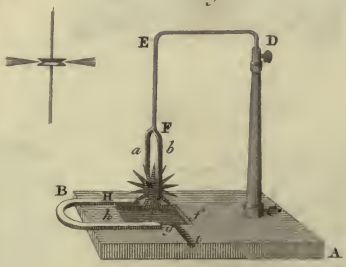

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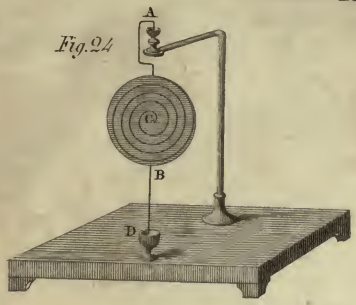
Fig. 24



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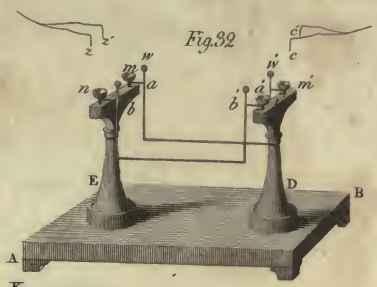

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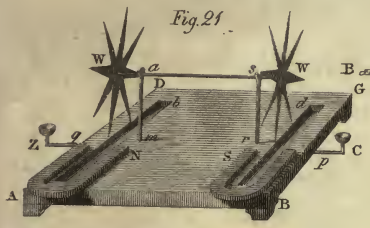

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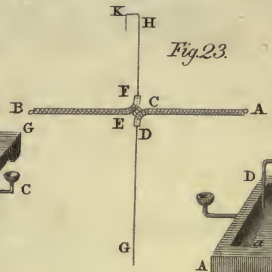

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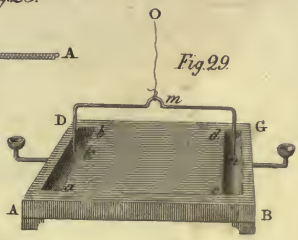

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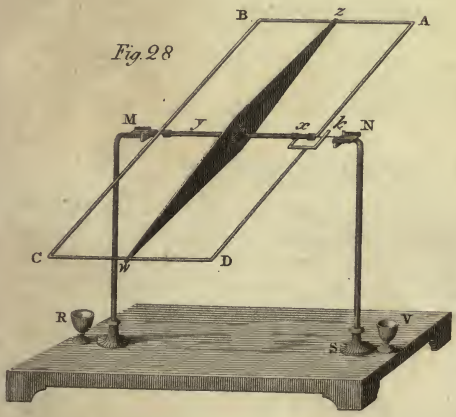
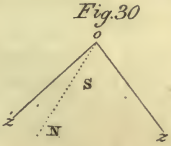


Fig. 31

Fig. 30



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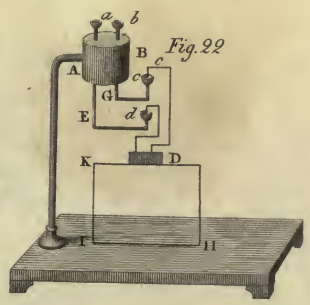

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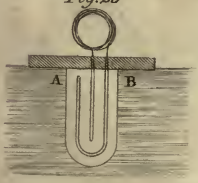

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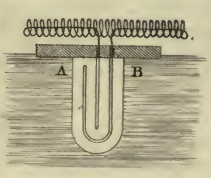

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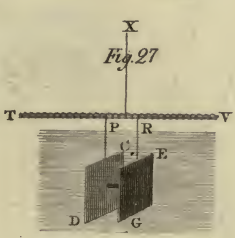

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"We have no hesitation in stating, that, as far as our knowledge of magnetism extends, all the laws which we have been describing are new facts in that science. By means of them we may compute, and by the most simple rules, the effect which a mass of iron will produce on the compass in any part of the world."—*Edinburgh Philosophical Journal, Oct.* 1821.

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* The Author was elected perpetual member of the Society, and honoured with their gold medal, and a complete set of the Society's Transactions.

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